ASSESSING THE NUMBER SENSE OF COMMUNITY COLLEGE DEVELOPMENTAL MATHEMATICS STUDENTS

by

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ABSTRACT

Students who enter community colleges unprepared for college-level mathematics often struggle to complete a college credential. To address this problem, college leaders and policymakers have employed various interventions that alter the structure of how underprepared students progress through their mathematics courses. For many students, this change to structure may simply shift, mask, or ignore the problem that they lack a strong foundation of mathematical understanding. This study approaches the problem by exploring what it means to effectively build mathematical knowledge upon a solid foundation. Specifically, this quantitative dissertation investigates the number sense of community college developmental mathematics students, how their number sense may improve throughout a typical developmental course, and if there is a relationship between their number sense and their success in that course.

A total of 165 students enrolled in developmental courses at Muskegon Community College in the fall of 2021 participated in the study. The students' number sense was measured using a 37-question multiple-choice number sense assessment. The assessment was administered as a pretest at the beginning of the semester and as a posttest at the end of the semester. Students' final course grades were acquired from the college's student records database. The data were analyzed using descriptive statistics, a one-way ANOVA test, paired samples *t* tests, and a Spearman correlation.

The findings of the study suggest that community college developmental mathematics students have poor number sense, there is a difference between the number sense of students enrolled in different developmental mathematics courses, students' number sense improves while completing some developmental mathematics courses but not others, and greater number sense is correlated with higher grades in some developmental mathematics courses.

KEY WORDS: number sense, community college, developmental mathematics

DEDICATION

To my dad, Jerry, who, despite struggling in his own high school math classes, always had number sense and helped instill it in me at a young age.

To my mom, Susan, who completed her own dissertation when I was in middle school, planting the seeds of intrigue that remained with me for the next 30 years.

To my wife, Lynn, whose unwavering support and encouragement were—without question—instrumental in my completion of this work.

To my son, Brian, who is my constant inspiration for improving myself in every way possible.

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CHAPTER 1: INTRODUCTION

Introduction

Developmental education, which is often associated with the reteaching of skills that were never mastered in high school, is a frequently debated topic amongst community college practitioners. More specifically, Ganga et al. (2018) define developmental education courses as those "designed to develop the reading, writing or math skills of students who are deemed—usually through standardized tests—underprepared for college-level courses" (p. 2). In the researcher's experience, the demand for developmental education is a product of such factors as the evolution of college entrance requirements, modifications to the K–12 curriculum, and the ever-changing demographics of college students. In the current climate, it is common to hear admonishments for the significant need for remediation in colleges and universities, but this reproach is nothing new. Since their inception, American universities have balanced their academic standards with their financial viability, often knowingly enrolling unprepared students to increase revenues (Bunner, 2018). According to Wyatt (1992), the majority of higher education institutions in the mid-to-late 1800s had preparatory departments designed to improve their students' basic reading and writing skills. Even in the early twentieth century it was common for many students enrolling at such institutions as Harvard, Yale, Princeton, and Columbia to not meet the entrance requirements (Bunner, 2018).

The proportion of underprepared students enrolling in college continued to increase throughout the twentieth century. With the passage of the GI Bill in 1944 and the 1965 Higher Education Act, access to college significantly expanded, particularly for students arriving from environments not typically conducive to college success (Arendale, 2002; Kimball, 2011). As community colleges emerged and evolved to meet the demands for access to higher education, various factors led them to becoming the primary institutions charged with teaching many underprepared students (Cohen et al., 2013). In the mid-to-late twentieth century, the number of college-aged students declined, which incentivized universities to recruit well-prepared students away from the community colleges; thus, the community colleges began serving a greater percentage of academically underprepared students (Cohen et al., 2013). Cohen et al. (2013) explain how phenomenon continued, noting that college entrance exam scores have a direct positive correlation with family income, and students coming from low-income homes are much more inclined to choose community colleges over universities. Henry and Stahl (2017) suggest that college leaders in the 1960s and 1970s favored a longer path of developmental courses to superficially diversify their student body, while keeping struggling students out of "real college classes where they were seen as not belonging" (p. 612). Cohen et al. (2013) support this notion, stating, "The pressure to allow anyone to enter a transfer program grew, the reason being that remedial programs were seen as catchalls for the less worthy, as holding tanks for students who would not succeed in higher education" (p. 260). The confluence of these factors helps explain the significant number of today's students at community colleges who are underprepared for college-level work.

In recent years, organizations like Complete College America, Achieving the Dream, the American Association of Community Colleges, Completion by Design, and Jobs for the Future have helped transform the mission of community colleges beyond just access to higher education to one focused on student completion, which is often measured by credential attainment (Kilgore & Wilson, 2017). This evolution has been accompanied by a greater focus on the role developmental education plays in advancing this completion agenda. Unfortunately, Jaggars and Stacey (2014) note that just 28% of community college students who take a developmental education course go on to earn a degree within eight years. The state of developmental mathematics in community colleges is particularly discouraging. According to a 2019 National Academies of Sciences, Engineering, and Medicine report, 59% of students from two-year institutions take developmental mathematics courses, of which fewer than 58% finish the developmental mathematics course sequence, and only 20% of those successfully complete a college-level mathematics course. This stark attrition is a barrier to college success that community college leaders from across the county have tried to address. The desire to improve students' academic success in developmental mathematics, and ultimately increase their rates of earning degrees, has led to many structural innovations including acceleration, placement modifications, corequisite remediation, modularization, redesigned academic pathways, and elimination of developmental coursework altogether (Rutschow et al., 2022). There is certainly a problem to be solved concerning both the lack of mathematical preparation for many students entering community college and the ability of institutions to guide those students to completion. This study examines the problem, not by addressing the systemic structure of developmental mathematics, but by focusing on the foundation of what it means for students to have a conceptual understanding of numbers and how this foundation might be related to their academic outcomes.

Description of Number Sense

There are several constructs that could be considered foundational to a student's conceptual understanding of numbers. Among them is *number sense*, a construct that has had little investigation at the community college level. The term number sense first became prominent in the mathematics education literature when it was introduced in the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 1989) as part of the recommended United States mathematics curriculum for Grades K–8. In response, a conference was organized with the goals of defining number sense, determining how it could be assessed, making recommendations of how it could be taught, and differentiating it from related ideas such as mental computation and computational estimation (Sowder & Schappelle, 1989).

Following the initial conference, many in the mathematical community continued the theoretical analysis of number sense. This began with Greeno (1991), who suggested that "number sense refers to several important but elusive capabilities, including flexible mental computation, numerical estimation, and quantitative judgment" (p. 170). McIntosh et al. (1992), in their development of the first significant framework, focused on the benefits of number sense with respect to life skills, describing it as "the basic number sense which is required by all adults regardless of their occupation and whose acquisition by all students should be a major goal of compulsory education" (p. 3). This reference highlights the importance of exploring adults' number sense, which is the focus of this dissertation. McIntosh et al. (1992) went on to propose a broader and more formal definition of number sense as "a propensity for and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information. It results in an expectation that numbers are useful and that mathematics has a certain regularity (makes sense)" (p. 4). Reys et al. (1999) explained number sense by proposing an outcome of its development:

> It results in a view of numbers as meaningful entities and the expectation that mathematical manipulations and outcomes should make sense. Those who view numbers in this way continually utilize a variety of internal "checks and balances" to judge the reasonableness of numerical outcomes. When an outcome conflicts with the perceived

> expectation, the person revisits the mathematical situation to externally view it, often through another lens, attempting to resolve the conflict. (p. 1)

Berch (2005) analyzed the various ways in which number sense had been described in the literature and observed that definitions had expanded to include "awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process, conceptual structure, or mental number line" (p. 1). Furthermore, Berch recommended extended work by the mathematical community to develop a more consistent definition that could be better operationalized. Supporting the belief that situational awareness is crucial to number sense, Maclellan (2012) stated, "Meaningful use of numerical information within an authentic context is the essence of Number Sense" (p. 4). Helmy et al. (2018) offered a less formal, but elegant description of number sense as "a kind of sensitivity or awareness to numbers" (p. 2). Most recently, Ghazali et al. (2021), through a systematic review of the themes found in the literature, developed an inductive definition of number sense as "analogous numerical cognition perceived via logical linkages and thinking skills through various communication modes" (p. 7).

In practice, the philosophy about what is important within mathematics education has evolved in recent decades from one that values and emphasizes written calculations and algorithms to one that recognizes the importance of a strong foundation in number sense, particularly in the primary and middle grades (NCTM, 1989, 2000). For example, following a standard procedure to correctly calculate 12/13 + 7/8 remains a skill to be learned, although it is now equally as important for a student to be able to suggest a reasonable estimate for this sum. A student equipped with a high level of number sense may recognize that because both 12/13 and 7/8 have a value slightly less than 1, their sum must be slightly less than 2. This precise question was asked on a foundational number sense study, and while 63% of 14-year-old students correctly calculated the sum, only 38% identified a correct estimate (McIntosh et al., 1997), suggesting more of these students possessed skills in written procedural calculations than in number sense.

Many studies, both in the United States and internationally, have indicated that students enrolled in compulsory education have low levels of number sense (Akkaya, 2016; Aperapar & Hoon, 2011; Bütüner, 2018; Facun & Nool, 2012; Gürefe et al., 2017; McIntosh et al., 1997; Menon, 2004; Mohamed & Johnny, 2011; Purnomo et al., 2014; Şengül & Gülbağcı, 2012; Singh, 2009; Singh et al., 2019; Yang, 2019; Yang et al., 2004, 2008; Yang & Lin, 2015; Yang & Sianturi, 2019, 2021). Furthermore, several studies of preservice teachers enrolled at universities have similarly suggested that their number sense levels are generally poor (Aktaş & Özdemir, 2017; Almeida et al., 2016; Hanson & Hogan, 2000; Tsao, 2005; Yaman, 2015; Yang, 2007; Yang et al., 2009). A lack of studies of community college students' number sense presents a notable gap in the literature. Given that completing a college-level mathematics course is a major barrier to the many college students who start in developmental coursework (National Academies of Sciences, Engineering, and Medicine, 2019), and number sense has been found to be correlated with mathematics achievement in other populations (Bütüner, 2018; Mohamed & Johnny, 2010; Singh et al., 2019; Yang et al., 2008), it is essential to understand the number sense of community college developmental mathematics students to better support them and improve their chances of academic success.

Related Constructs

Within the literature, several related terms are sometimes used interchangeably with number sense and with each other. However, these remain distinctly different constructs, and for the purposes of this study, it is necessary to describe their similarities and differences. First, *number flexibility* or *flexible computation* refers to one's ability to deconstruct numbers and use the parts to complete a mathematical process. Greeno (1991) describes flexible computation as the "recognition of equivalence among objects that are decomposed and recombined in different ways" (p. 193). One might exhibit number flexibility or flexible computation in calculating 23 × 12. Rather than using a standard algorithm, one could mentally deconstruct the 12 into 10 + 2, compute 23 × 10 and 23 × 2, and then add the two resulting products. Gray and Tall (1994) attend to this ability using the idea of a *procept*, which they define as the "amalgam of concept and process represented by the same symbol" (p. 6). An example of a procept would be the + symbol indicating both the process of addition (such as finding a strategy to add 3 + 2) and the concept of sum, which is the result of addition (3 + 2 = 5). Students with the ability to seamlessly navigate between process and concept demonstrate number flexibility. Gray and Tall suggest that the proceptual thinker is able to manipulate prior knowledge to derive new knowledge, while the nonproceptual thinker is only able to stack complex processes, further increasing the difficulty of learning mathematics as they encounter additional challenges. This ability to work flexibly with numbers and computations is often considered a component of number sense (Akkaya, 2016; Aktaş & Özdemir, 2017; Gürefe et al., 2017; McIntosh et al., 1992; Tsao, 2004; Ulusoy, 2020; Yaman, 2015), suggesting number sense is a broader construct.

The terms *quantitative literacy* and *numeracy* are often used interchangeably in the literature, although some authors see distinctions between them. According to Maclellan (2012), quantitative literacy is the ability to apply quantitative knowledge and reasoning in the appropriate situation. Numeracy, according to Tout (2020), refers to cognitive elements, such as one's quantitative knowledge base and skills, as well as non-cognitive elements, such as attitudes and beliefs. Furthermore, numeracy is often connected to an adult's ability to reason critically across a broad range of mathematical situations (Tout, 2020). Vacher (2014) suggests that numeracy and quantitative literacy are often used synonymously within the literature, while also having multiple meanings. Both numeracy and quantitative literacy remain relevant to the purpose of this study as they have been demonstrated to be dependent upon one's ability to develop number sense (Maclellan, 2012).

Finally, number sense itself is a term used inconsistently in the literature. I. Whitacre et al. (2020) completed the most recent and most thorough review of the existing number sense literature to examine how the term has been used as a construct. The authors conclude that there are three distinct number sense constructs within the literature—*approximate number sense*, *early number sense*, and *mature number sense*—each with its own distinct characteristics (Whitacre et al., 2020). Approximate number sense, as described by Dehaene (1997, 2011), is primarily explored in psychological research and refers to the concept of subitizing, which is the ability to instantly identify the cardinality of a set of objects. Unlike approximate number sense, both early number sense and mature number sense are generally researched by those in the field of mathematics education. However, early number sense is typically associated with students in the grade ranges of preschool through second grade and is comprised of such skills as "number recognition, counting, number patterns, number comparisons, number operations, and estimation" (Whitacre et al., 2020, p. 101). The characteristics of mature number sense attended to by the literature are that it is learned, involves habits of mind, is studied in populations ranging from elementary students to adults, and is measured with tests that align with various components of number sense (Whitacre et al., 2020).

Theoretical Framework

According to Berch (2005), researchers have used at least 30 different features to describe and measure number sense, based on the population being studied or the specific research questions. For the purpose of this study, I. Whitacre et al.'s (2020) description of mature number sense will be used as a basis for the theoretical framework as it aligns most closely with the population being studied and the purpose of the research. As described by Whitacre et al. (2020), mature number sense is learned, involves habits of mind, generally involves understanding of multidigit and rational numbers, and is studied in populations ranging from elementary students to adults. Each of these characteristics is directly related to the population of community college developmental mathematics students and their learning environment. Additionally, Whitacre et al. (2020) note that mature number sense is measured using tests that align with various components of number sense. For this study, the components used are informed by Hsu et al. (2001, as cited in Whitacre, 2012):

1. Understanding number meanings and relationships
2. Recognizing the magnitude of numbers
3. Understanding the relative effect of operations on numbers
4. Developing computational strategies and being able to judge their reasonableness
5. Ability to represent numbers in multiple ways. (p. 28)

These or similar components have been used in other studies that measure the number sense of college students (Whitacre, 2012; Whitacre & Nickerson, 2016). A brief description of each component follows.

Understanding Number Meanings and Relationships

Understanding number meanings and relationships indicates that students have a strong understanding of the base ten number system; thoroughly comprehend place values; grasp the meaning of and relationships between whole numbers, fractions, and decimals; and can use multiple ways to represent numbers (Li & Yang, 2010; McIntosh et al., 1992; Reys & Yang, 1998; Yang, 2019; Yang et al., 2004; Yang & Lin, 2015; Yang & Sianturi, 2019; Yang & Tsai, 2010). For example, students should comprehend that there are an infinite number of values between 4/7 and 5/7 or between 9.43 and 9.44, and they should recognize that 0.5 x 840 is the same as 840 ÷ 2. As clarified by Yang and Sianturi (2021), it is important to note that this first component is often the basic knowledge necessary for the others and should not be considered as a component independent from the rest.

Recognizing the Magnitude of Numbers

Recognizing the magnitude of numbers requires students to grasp the absolute size of a number as well as its size relative to other numbers (Mohamed & Johnny, 2011; Tsao, 2005). Additionally, students should be able to generate meaningful ways to compare two fractions, such as noticing the same numerators, comparing them to a benchmark such as 1/2, or comparing the complement of each to the whole (Li & Yang, 2010; Yang, 2007, 2019; Yang et al., 2004; Yang & Lin, 2015; Yang & Sianturi, 2019, 2021; Yang & Tsai, 2010). For example, students could demonstrate number sense in this component by identifying the greater value when comparing 7/13 and 7/15, not by getting a common denominator, an approach traditionally taught in many schools, but by noticing that 7/13 is slightly greater than 1/2 and 7/15 is slightly less than 1/2.

Understanding the Relative Effect of Operations on Numbers

Students demonstrate an understanding of the relative effect of operations on numbers by recognizing how each of the four basic operations, in relationship to the values of the numbers themselves, affects the result (Mohamed & Johnny, 2011). This includes knowing that multiplication does not always yield a larger number, nor division a smaller number (Yang, 2019; Yang et al., 2004; Yang & Lin, 2015; Yang & Tsai, 2010). For example, students should sense that the sum of 3/7 and 1.5 is slightly less than 2. Similarly, they should recognize that 487 ÷ 0.99 is slightly greater than 487 because the divisor is slightly less than 1.

Developing Computational Strategies and Being Able to Judge Their Reasonableness

When faced with computational problems, students must have the ability to mentally apply multiple strategies and judge the reasonableness of the results (McIntosh et al., 1992; Yang et al., 2008; Yang & Sianturi, 2019; Yang & Tsai, 2010). They should also be able to make reasonable estimates without relying on written calculations or algorithms (Li & Yang, 2010; Mohamed & Johnny, 2011; Yang & Sianturi, 2019, 2021; Yang & Tsai, 2010), including making judgments about applied, real-world scenarios (Li & Yang, 2010). For example, given several options, students could deduce the closest number to the product of 18 × 19 without calculating exactly. Similarly, if given the product of 103 × 236, they could determine the product of 103 × 235 without using a standard algorithm. Additionally, given a number line with endpoints 0 and 0.1, they should be able to determine the value of the midpoint.

Ability to Represent Numbers in Multiple Ways

Students must have the ability to represent numbers in multiple ways, such as through symbols, pictures, and number lines (Yang, 2019; Yang & Huang, 2004; Yang & Sianturi, 2021; Yang & Tsai, 2010), and a strong comprehension of the concepts on which these multiple representations are based (McIntosh et al., 1992; Yang et al., 2008; Yang & Sianturi, 2021). Additionally, students must have the capacity to switch between and determine the most appropriate representations, such as a fraction, decimal, or percentage, based on the given mathematical situation (McIntosh et al., 1992; Yang & Sianturi, 2019). For example, students should recognize that 1 + 1/4 is equivalent to 5/4, 1.25, and 125% and know when to use each representation appropriately. They could also identify the approximate location of a value, such as 2.19, on a number line, given the points 0, 1, 2, and 3.

Given that number sense is foundational to the mathematical development of students of all ages, this theoretical framework will apply the components of number sense to address the following problem.

Problem Statement

The problem this study is addressing is that students who enter community colleges unprepared for college-level mathematics are much less likely to complete a college degree than their mathematically-prepared counterparts (Burley et al., 2009). Many organizations have researched this problem and attempted to implement innovative structural modifications, most of which have had little to moderate impact (Rutschow et al., 2022). Despite this national attention and significant resources dedicated to developing structural interventions, little has been done to investigate what students' foundational mathematical deficiencies may be and how they can be more effectively addressed at a classroom level (Cox, 2018).

Purpose and Significance of the Study

The purpose of this study is to better understand the number sense of community college developmental mathematics students, how their number sense may improve throughout a typical developmental mathematics course, and if there is a relationship between their number sense and their success in that course.

For all learners of mathematics, number sense is a crucial foundational skill (Maclellan, 2012; NCTM, 1989, 2000). However, the majority of recent interventions directed at improving community college developmental mathematics outcomes have been structural in nature and ignored the content of developmental mathematics courses and the manner in which the content is taught (Rutschow et al., 2022). While some structural interventions may occasionally improve pass rates, for students with fractured mathematical foundations, these interventions may also mask an underlying problem. An investigation aimed at studying students' conceptual development may shift the focus away from structural interventions and toward an approach that ensures students are building mathematical knowledge upon a solid foundation of number sense.

For adults, number sense is essential to developing quantitative literacy (Maclellan, 2012), a skill crucial for navigating quantitative situations that arise in everyday life. However, most of the studies of number sense have been conducted with children at the K–12 level and not with adult populations. There have been several studies assessing the number sense of college students, but those have primarily been conducted at universities with preservice teachers (Aktaş & Özdemir, 2017; Almeida et al., 2016; Hanson & Hogan, 2000; Tsao, 2004, 2005; Yang, 2007; Yang et al., 2009). Few researchers have investigated the number sense of community college students. Steinke's (2017) work, although it was with community college developmental mathematics students, was limited to analyzing students' sense of quantity by plotting certain numbers on a blank number line. Stigler et al. (2010) and Givvin et al. (2011) studied the conceptual understanding that community college developmental mathematics students have, but their research did not incorporate the theoretical framework common in the number sense literature. Thus, there is a clear lack of research regarding the number sense of community college developmental mathematics students, and the findings of this study will begin to fill this gap in the existing knowledge base.

Research Questions and Hypotheses

To investigate the problem, this study explored the following research questions:

6. What level of number sense do community college developmental mathematics students have?

7. Is there a difference between the number sense of students enrolled in different developmental mathematics courses?

8. Does completing a traditional community college developmental mathematics course improve students' number sense?

9. Do students with greater number sense have better academic success in community college developmental mathematics courses?

Research Question 1 was examined using descriptive quantitative statistics. Research Question 2 was examined using inferential statistics. The null hypothesis for Research Question 2 was:

> H_0: There is no significant difference between the number sense of students who are enrolled in different developmental mathematics courses.

This was tested against the following alternative hypothesis:

> H_a: There is a significant difference between the number sense of students who are enrolled in different developmental mathematics courses.

Research Question 3 was examined using inferential statistics. The null hypothesis for Research Question 3 was:

> H_0: As a result of completing a traditional community college developmental mathematics course, there will be no significant increase in students' number sense.

This was tested against the following alternative hypothesis:

> H_a: As a result of completing a traditional community college developmental mathematics course, there will be a significant increase in students' number sense.

Research Question 4 was explored using inferential statistics. The null hypothesis for Research Question 4 was:

> H_0: There is no significant correlation between students' number sense and their success in community college developmental mathematics courses.

This was tested against the following alternative hypothesis:

> H_a: A significant, positive correlation exists between students' number sense and their success in community college developmental mathematics courses.

Overview of the Study

This quantitative study was designed to determine the level of number sense of community college developmental mathematics students, if there was a difference between the number sense of students in different developmental courses, if their number sense improved throughout their time in the developmental courses, and if their academic success in their mathematics developmental courses was related to their levels of number sense. The study was conducted at a medium-sized midwestern community college in the fall of 2021 with students who were enrolled in one of four developmental mathematics courses. The students were invited to participate in a pretest and posttest at the beginning and end of their mathematics courses and share their final course grades with the researcher. The number sense assessment used was an adapted version of the Number Sense Rating Scale, originally created by Hsu et al. (2001, as cited in Whitacre, 2012), that has been used in various settings and countries with age groups ranging from elementary school children through preservice teachers in college (Whitacre, 2012; Whitacre & Nickerson, 2016; Yang, 2003).

Delimitations and Limitations

1. To limit the scope of the research, several delimitations were placed upon the design of this study. These delimitations included:

    - The dependent variable for Research Question 4 was limited to fall 2021 mathematics course grades.

    - Data were collected from only one educational institution, a medium-sized community college in the midwestern United States.

    - Students' number sense was tested using a single quantitative instrument. Such multiple-choice tests can only examine students' abilities to identify correct answers within a given time constraint. While the questions were designed to capture students' capacity to utilize number sense strategies as opposed to traditional written calculations, an incorrect response does not imply they were not using number sense approaches (Yang & Lin, 2015).

- Follow-up interviews were not conducted and could have revealed a deeper understanding of students' thinking.
- Only students 18 years of age and older were allowed to participate.
- Only students enrolled in synchronous classes were invited to participate, which excluded one section of 40 students enrolled in an asynchronous online Math 040 (Beginning Algebra) class.
- Factors such as age, years since last mathematics course, gender, income level, instructor, and length of course were not considered.
- Non-academic factors that can affect students' grades such as childcare, work schedules, motivation, adherence to academic integrity, or health were not considered.

2. Additionally, several unintended limitations occurred throughout the research process, restricting the ability of the results to be generalized to other institutions and other groups of students. These limitations included:

   - Due to COVID-19 restrictions limiting the college's capacity to administer placement tests, students were allowed to self-place in the fall semester of 2021, bypassing any prerequisites if they chose to do so. Thus, there was no mandated standard in place for assigning students to different levels of mathematics. Students were provided guidance through academic counseling and on the college's website (Muskegon Community College, 2019, 2021b).
   - For consistency, students participating on campus and students participating synchronously online were both provided the same online testing instrument. The instructions indicated that students should answer questions by thinking through the problems and not by using a calculator, a pencil and paper, or other aids in calculation. However, there is no way for the researcher to confirm that the students participating synchronously online did not use such resources.
   - Although all students were asked to complete the pretest and posttest for use in the college's general assessment reports, students were given the opportunity to have their results excluded from this research study. There may have been factors that are correlated with number sense, such as mathematical confidence, that contributed to students' decisions to have their results included in the study.
   - Some students were absent on the day that either the pretest or posttest was given, and some students dropped their course during the semester or received an incomplete. Specifically, out of the 271 students enrolled in a developmental mathematics course to begin the semester, 165 (61%) participated in the pretest phase of the study. Of those 165 students, 148 (90%) earned a final letter grade in their course and 100 (61%) participated in the posttest phase.

Definitions

To ensure clarity and consistency, a list of key terms and their definitions as they relate to this study are provided below:

- *Asynchronous*—an online class offered in a format with no scheduled meeting times.
- *College-level*—coursework that generally earns students credit toward a post-secondary credential. College-level courses are typically numbered as 100-level or higher.
- *Developmental/Remedial*—coursework that may be mandated as a prerequisite for another course based on a student's college entrance scores or another similar measure. In general, developmental courses are numbered lower than 100, address topics classified as basic skills, and do not earn a student credit toward a credential.
- *Number sense*—the capacity to understand number meanings and relationships, recognize the magnitude of numbers, understand the relative effect of operations on numbers, develop computational strategies and judge their reasonableness, and represent numbers in multiple ways (Hsu et al., 2001, as cited in Whitacre, 2012).
- *Synchronous*—a class with scheduled meeting times, either virtual or in person.

Assumptions

The following assumptions were necessary in the conduct of this research:

- Although students were able to self-place into their mathematics courses, they did so accurately.
- The instrument used in this study appropriately measures students' number sense.
- Students took the number sense assessment seriously and performed to the best of their abilities.
- Students did not use any writing utensils or calculators while answering the questions on the number sense assessment. The time limit set for individual questions was intended to restrict this opportunity.
- Students participating in a synchronous online class from a remote location performed similarly to those students participating in a traditional, face-to-face class on campus, both in their coursework as well as on the number sense assessment.
- The administrator of the number sense assessment delivered the instructions correctly and consistently to all classes.

Summary

Students who place and enroll in developmental mathematics courses at a community college are less likely to persist toward completing a college degree than those who arrive fully prepared for college-level coursework (Burley et al., 2009). Although this problem has received considerable attention in recent years, most of the approaches to improving student outcomes have focused on changing institutional structures. The purpose of this study is to explore the problem by investigating the foundational mathematical understanding that these students possess, specifically by examining their number sense and how it may be associated with their academic success.

Chapter 2 reviews the significant literature concerning developmental mathematics at community colleges as well as number sense. Chapter 3 describes the research design and methodology used to answer the research questions. Chapter 4 presents the results of the study, the data that were gathered, and analyses of how those data answer the research questions. Finally, Chapter 5 discusses implications of the findings and offers recommendations for future research.

CHAPTER 2: LITERATURE REVIEW

Introduction

The purpose of this study is to better understand the number sense of community college developmental mathematics students, how their number sense may improve throughout a typical developmental mathematics course, and if there is a relationship between their number sense and their success in that course. To properly frame the purpose of the current study, this literature review has been organized into five sections. The first section highlights the efficacy of developmental mathematics in community colleges, including the influence that traditional developmental mathematics courses may have on student outcomes as well as the factors that may contribute to students' success in developmental mathematics courses. The second section explores many of the recent interventions and innovations that have been employed to improve the academic success of developmental mathematics students. The third section introduces the construct of number sense and describes the foundational studies upon which the current number sense framework is based. The fourth section examines research that has been conducted on the number sense of children and adolescents. The fifth and final section discusses the research that has investigated the number sense of college students. These final two sections include studies that explored such topics as understanding students' number sense, interventions to improve students' number sense, and how number sense may be related to students' academic achievement.

Developmental Mathematics in Community Colleges

There has been a considerable amount of research—sometimes contradictory—examining the relationship between developmental mathematics and student outcomes. Some researchers have found that developmental mathematics has a negative impact on students' likelihood of degree attainment while others have demonstrated its benefits. In addition to investigating the effectiveness of developmental mathematics, other studies have focused on the factors that may lead to students' success in developmental mathematics courses. This section explores the literature addressing both issues.

The Effect of Developmental Mathematics on Students' Academic Success

Several studies have investigated the effect that enrollment in developmental mathematics has on various measures of students' academic success. In a study of long-term outcomes, Burley et al. (2009) explored bachelor's degree attainment rates for developmental mathematics students using records from 6,832 postsecondary students in the National Educational Longitudinal Study: 88/2000 dataset. The purpose of Burley et al.'s work was to predict the likelihood of developmental mathematics students to earn bachelor's degrees. Burley et al. found that students referred to developmental mathematics were less likely to earn a degree than those who were not. Burley et al. also reported that the students' mathematical skills, socioeconomic status, and locus of control accounted for 24% of the variance of degree attainment between students who did or did not need mathematics remediation.

Fong et al. (2015) utilized data from eight large urban community colleges in California. The researchers completed a thorough examination of 62,082 students' persistence and success rates in the developmental mathematics course sequence. Fong et al. observed that placement into developmental mathematics courses was generally not a discouragement for students to enroll in those courses, and students who entered developmental mathematics at lower levels attempted and passed their courses at the same rates as those who placed at higher levels. Additionally, while the overall rate of progression through all the developmental mathematics courses was low, those students who did persist through the entire sequence often had greater success in higher courses than the students who placed directly into those higher courses (Fong et al., 2015).

In another large-scale study, Xu and Dadgar (2018) used transcript data for 24,664 first-time community college students in the Virginia Community College system to examine the effectiveness of remediation on the students with the lowest mathematics skills. By comparing students with similar academic skills, Xu and Dadgar found that longer sequences of developmental mathematics generally reduce the likelihood that students with the lowest skills will earn a degree.

Cox and Dougherty (2019) took a different approach by exploring the goals that developmental mathematics students and their instructors had for the course, the extent to which the students accomplished those goals, and how the students' perspectives with respect to learning complicate the way in which completion rates are generally used as an indicator of success. Cox and Dougherty (2019) discovered that the primary goals of both students and instructors were for the students to gain greater confidence in their mathematical abilities and an improved relationship with mathematics. However, few students reported reaching this goal, despite completing the course successfully. The authors suggested their findings revealed a disconnect between the perceived purpose of developmental mathematics courses, the way in which students are assessed, the instructional approaches, and completion rates (Cox & Dougherty, 2019).

Factors Influencing Students' Success in Developmental Mathematics

Rather than examining the effect that developmental mathematics may have on students' academic success, some researchers have explored the factors that may contribute to the success of developmental mathematics students. Smith et al. (1996) conducted an ethnographic observation of 218 students enrolled in two developmental mathematics courses at a university in the United States. Based on their observations, Smith et al. found a significant positive correlation between students' grades and their observed engagement in class activities. Pruett and Absher (2015) explored developmental education outcomes in general—not just specifically developmental mathematics—and although their work came nearly 20 years after Smith et al.'s, the findings were similar. Using data from the 2013 Community College Survey of Student Engagement, Pruett and Absher examined responses from 23,665 developmental education students at 718 United States institutions to expose factors that may contribute to the retention of students enrolled in developmental education courses. Pruett and Absher observed that other than a student's cumulative college grade point average, devotion to studies had the greatest correlation with retention.

Using a qualitative approach, Cafarella (2014) conducted interviews with developmental mathematics faculty at an urban community college in the midwestern United States. Cafarella (2014) explored both inhibitors of students' academic success and approaches to teaching that fostered academic success. Cafarella's (2014) subjects noted factors that impede academic success generally include extremely low computational skills, calculator dependency, poor attendance, a lack of work ethic, and excessive external obligations. Conversely, the teaching practices Cafarella (2014) found to be most effective were clear communication with students, organization and structure within the courses, collaborative learning, and frequent low-stakes assessments.

Cox (2015) examined the instructional approaches in two large, urban-serving community colleges to better understand what students were experiencing in developmental mathematics courses and how those instructional approaches affected their course success. Cox (2015) observed that some instructors employed traditional approaches, such as emphasizing terminology and practicing algorithms, while others stressed mathematical discourse, problem solving, and activities designed to develop conceptual understanding. The instructional approaches of the latter were associated with better pass rates in those courses than of the former (Cox, 2015).

Fong et al. (2015), whose work was discussed in the section on the effect of developmental mathematics on students' academic success, also examined individual factors that contribute to students' success in developmental mathematics. Fong et al. observed that female students generally are more successful than males, African American students are generally less successful than White students, and Latino students, when compared to other groups, have relatively high persistence rates but low passing rates. Fong et al. noted that their findings suggested environmental pull factors, such as family or work obligations, decrease the likelihood of success in developmental mathematics, particularly for Latino students.

At a medium-sized community college in Texas, Acosta et al. (2016) studied 290 developmental mathematics students to determine the effect of various factors on their success in subsequent college-level courses. Acosta et al.'s findings suggested that delivery modality (comparing face-to-face and online students) and time since high school had no impact on the students' likelihood to pass the college-level mathematics course. However, the students' cumulative college grade point average prior to taking that college-level course was found to be a significant predictor of success (Acosta et al., 2016), similar to Pruett and Absher's (2015) finding that college grade point average was predictive of retention.

Improving Developmental Mathematics Outcomes

As noted in the previous section, the effectiveness of developmental mathematics courses in facilitating students' academic success is questionable. In response, colleges have adopted various interventions to improve the success of students who begin their college experience in developmental mathematics courses. This section discusses research on several of the most common strategies that have been implemented in recent years.

Mindset Interventions

A number of researchers have explored the effect that addressing students' non-cognitive factors, such as behaviors, habits, and attitudes, can have on their academic performance. Specifically, mindset interventions attempt to change students' understanding of success from one of a fixed mindset to one of a growth mindset. That is, mindset interventions are designed to teach students that their level of intelligence is not fixed and their abilities can improve through hard work (Dweck, 2006). Research in the K–12 setting has illustrated the positive effect that mindset interventions can have on students' learning (Paunesku et al., 2015). At the community college level, researchers have begun replicating such approaches to determine if these interventions can have a similar effect. Mills and Mills (2018) conducted a study with developmental mathematics students at a small liberal arts college with a majority of first-generation college students, a population similar to that of many community colleges. The students experienced an intervention designed to improve their growth mindset and the researchers observed higher grades from those who either began with a high growth mindset or received a treatment to improve their growth mindset (Mills & Mills, 2018).

A report by the Center for Community College Student Engagement (2019) found more community college students have a fixed mindset for mathematics than for English or general intelligence. The report also indicated that higher college grade point averages are correlated with a growth mindset, greater self-efficacy, and a stronger sense of belonging, suggesting that interventions to improve these psychological factors may be of particular importance in improving student outcomes (Center for Community College Student Engagement, 2019).

Samuel and Warner (2021) conducted an intervention to improve the mindfulness and growth mindset of developmental statistics students at a community college. Their approach led to reduced mathematics anxiety and increased mathematics self-efficacy, although the authors did not examine the effects these improvements had on the students' short- or long-term outcomes (Samuel & Warner, 2021).

Acceleration

Acceleration has become a common strategy to address poor long-term success of students who place into developmental mathematics courses, particularly for those facing a multi-course sequence prior to enrolling in college-level coursework. These accelerated courses were created under the assumption that because there are points between semesters during which students can decide to not reenroll, students are more likely to persist if colleges reduce the number of these exit points prior to the students earning college-level credits (Jaggars et al., 2014). Acceleration models can take a variety of forms to shorten the time that students need to spend completing developmental courses, but a common structure is a paired-course model. In paired courses, students complete two developmental courses in a single semester sequentially with each course meeting twice as many hours per week and lasting just half the length of the standard semester. Research around acceleration models has attempted to determine the impact they have on persistence, completion of subsequent college-level coursework, credential completion, and retention of course content.

Hodara and Jaggars (2014) studied students enrolled in developmental mathematics courses at six City University of New York community colleges between 2001 and 2007. Students enrolled in a shorter sequence were more likely to enroll in and complete college-level mathematics coursework than those in the longer sequences (Hodara & Jaggars, 2014). However, the shorter accelerated model had a limited effect on credential attainment as students enrolled in the shorter sequence were just one percentage point more likely to earn an associate degree over three years (Hodara & Jaggars, 2014).

The Community College of Denver created a FastStart program, which employed the paired-course approach along with additional counseling for the enrolled students and encouragement to register for a student success course (Jaggars et al., 2015). Jaggars et al. (2015) found that over three years, FastStart students were 11 percentage points more likely to complete a college-level mathematics course than those in the traditional sequence, a result that was fueled by their greater propensity to enroll in the subsequent college-level course.

Approaching the research from a different perspective, Cafarella (2016) conducted a qualitative study, interviewing six faculty from three midwestern community colleges who had experience teaching developmental mathematics in an accelerated format. Cafarella's (2016) findings suggested that while acceleration strategies can be effective, it is an approach that is not a fit for all students. The interviewed faculty believed that a student’s likelihood of success in an accelerated developmental mathematics course is dependent upon their comfort level with computer software, having a greater initial skill level, and the instructor's comfort level with teaching within the accelerated structure (Cafarella, 2016).

Redesign of Placement Policies

Higher education institutions, including community colleges, have traditionally used results of standardized placement tests, such as ACCUPLACER and Compass, to assign students to various levels of developmental mathematics. The Community College Research Center conducted two large-scale studies (Bailey et al., 2010; Scott-Clayton et al., 2014) to analyze the effectiveness of these standardized tests at placing students into the proper developmental mathematics courses. Bailey et al. (2010) observed that very few students ever completed the developmental mathematics sequence and nearly one-third never enrolled in the developmental course to which they were assigned. Furthermore, Scott-Clayton et al. (2014) found that many students were misplaced when the traditional standardized tests were used, with far more students being under-placed than over-placed. Scott-Clayton and Stacey (2015) furthered Bailey et al.'s (2010) and Scott-Clayton et al.'s (2014) research by incorporating information about the students' high school performance and demographics. Using statistical model simulations, Scott-Clayton and Stacey (2015) concluded that using high school grade point averages would have led to fewer misplacements and greater success rates in college-level courses.

Some community colleges have begun adopting a more holistic approach to placement by factoring in non-cognitive measures unrelated to skill in mathematics—including motivation, time management skills, and support systems—in addition to their standardized test scores. Ngo et al. (2018) studied a large urban community college district in California that had implemented this approach, often referred to as incorporating *multiple measures*. In Ngo et al.'s study, the college allowed students whose placement scores were below the traditional cutoff point to enroll in a higher mathematics course if they demonstrated non-cognitive strengths. Ngo et al. observed that these students were just as likely to pass their mathematics courses as those who had higher placement test scores. Their findings suggested that approximately one quarter of the students who were traditionally placed using only the standardized tests may have been incorrectly assigned (Ngo et al., 2018), and, similar to the findings of Scott-Clayton et al. (2014), most of those were under-placements.

Seven community colleges in the State University of New York system participated in a study of multiple measures placement conducted by Barnett et al. (2018). Using a random assignment study, Barnett et al. used students' high school grade point averages, years since graduation, standardized placement test scores, and their performance in college-level mathematic courses to design an alternative placement system that weights different student characteristics. While the creation of the new system proved more complex than anticipated, Barnett et al. found that it resulted in a higher rate of placement, enrollment, and success in college-level mathematics courses.

Other colleges have empowered students further by allowing them to select their own starting level of mathematics based on the college's recommendation, often referred to as *guided* or *directed self-placement*. Kosiewicz and Ngo (2020) used data from a large urban community college district to study the results of guided self-placement in mathematics. One of the colleges in the district had unintentionally let their standardized test license expire and consequentially allowed a cohort of students to choose their own mathematics courses, creating a natural experiment. Kosiewicz and Ngo observed that while self-placement generally led to positive outcomes, the gains were primarily for students who identified as White, Asian, or male; there were no or reduced benefits for female, Black, and Hispanic students. The authors attributed these results to self-determination theory and stereotype threat, which may have led those populations to underestimate their own abilities and place themselves at a level lower than may have been appropriate.

Corequisite Remediation

Similar to acceleration strategies, corequisite remediation is an approach that strives to help students advance to college-level coursework more quickly than the traditional prerequisite model. The corequisite approach differs in that students—often those testing near the margin of placement cutoff scores—are placed directly into a college-level course along with a corequisite developmental course. This corequisite course is designed to provide students with additional content support, and the structure allows students to earn their college-level mathematics credits immediately.

In 2014, the state of Florida implemented a policy that made all developmental education courses optional for community college students. The corequisite model was adopted by some schools as an alternative option for students who chose to immediately enroll in college-level mathematics coursework but still desired some supplemental developmental support. Using the Florida Education Data Warehouse, Park et al. (2018) studied records from over 20,000 students who would have previously been required to take developmental mathematics courses. Park et al. found that while only 3.4% of students chose the corequisite model, they were significantly more likely to succeed in their college-level courses than those who did not take the corequisite support course.

Ran and Lin (2019) examined data from 13 community colleges in Tennessee to compare the effects of placement into corequisite remediation with those of traditional remediation and direct placement into college-level courses. Their findings suggested that students who scored near the placement cutoff scores were more likely to pass their college-level mathematics courses in a corequisite approach than in a traditional remediation model, and they were just as likely to pass as those who placed directly into college-level courses (Ran & Lin, 2019). There were, however, no significant effects of the corequisite approach on persistence, university transfer, or completion (Ran & Lin, 2019). Logue et al. (2019) found a similar impact of corequisite remediation in a long-term evaluation at three New York community colleges, although their results also indicated significantly higher graduation rates.

Although the corequisite approach is promising, research in this area is limited, and the studies mentioned were restricted in the populations they analyzed. Park et al.'s (2018) work was on a large scale, but because the corequisite model was new at the time, only 3.4% of the observed students were enrolled in such a structure. The study by Ran and Lin (2019) focused only on students near placement cutoff scores, and the Tennessee statewide reform had also integrated efforts to align students' mathematics coursework with their degree paths. Similarly, Logue et al. (2019) only studied students who had placed into an elementary algebra course and were pursuing a degree path that required statistics as the college-level course.

Developmental Mathematics Redesign

With respect to developmental mathematics, the term *redesign* is generally associated with a complete overhaul of the existing model. While the interventions previously discussed in this section are sometimes implemented individually, a redesign may include any number of these interventions as well as other changes to instructional delivery, curriculum, assessments, student interactions, and supplemental support. Although these redesign efforts generally appear to positively affect students' academic success in developmental mathematics, Cafarella's (2016) literature review highlighted several challenges of redesign that can hinder student success.

Cafarella (2016) observed that some developmental mathematics programs are in a constant state of redesign as their institutions are constantly searching for the one structure that will yield success, an approach that can lead to burnout and poor morale amongst faculty as well as increased costs. Additionally, Cafarella (2016) observed that without careful planning and structure, redesign efforts can create inconsistent and chaotic learning environments.

Bishop et al. (2018) studied the effect of North Carolina's developmental mathematics redesign in which the structure was condensed from three 16-week courses to eight 4-week courses. North Carolina's approach also integrated features of a modularized structure, in that the curriculum was broken into more narrow concepts, students could retake a course in the same semester if necessary, and each course had a mastery learning requirement (Bishop et al., 2018). Bishop et al. analyzed the subsequent college-level mathematics course pass rates at 12 of North Carolina's 58 community colleges and found that there was no difference in the success of students in the different course lengths, as those in the compressed format completed the college-level course at the same rate as those in the traditional structure.

Another form of redesign that has shown significant promise in recent years is *guided pathways*. The concept of guided pathways, popularized by Bailey et al. (2015), centers around simplifying students' college experience to ensure they are completing the courses that most efficiently lead to the appropriate credential for their career goals. While this model is not exclusively an intervention designed to improve developmental mathematics, redesigning the developmental education structure is a key component to fully implementing guided pathways. Most colleges have traditionally aligned their developmental mathematics curriculum with the content necessary for success in college-level algebra. Bailey et al. (2015) asserted that this model is detrimental to the many students whose programs require a quantitative reasoning or statistics course in lieu of a traditional college algebra course. Thus, colleges that have adopted a guided pathways approach have redesigned their developmental mathematics curriculum to include prerequisite courses for quantitative reasoning and statistics. Rutschow et al. (2019) reported on the results of a mathematics pathways impact study facilitated by the Dana Center. In the Dana Center study, which was conducted at four Texas community colleges, students were randomly assigned to either the two traditional algebra-based developmental mathematics courses or a single developmental course aligned with their mathematics pathway. Rutschow et al. (2019) observed that students in the redesigned pathways course were more likely to complete their developmental mathematics sequence, more likely to pass a college-level mathematics course, and able to acquire more college-level mathematics credits in their first three semesters. Additionally, Rutschow et al. (2019) noted that the greatest impacts were realized by those students who performed lowest on the standardized placement tests.

Foundations of the Number Sense Construct

As discussed, there have been numerous efforts to enhance the academic success of community college developmental mathematics students, most of which have addressed the structure and design of the developmental mathematics courses or the related student support systems. In contrast, the current study is focused on exploring the foundational mathematical understanding of this population, particularly their number sense. There have been myriad approaches used to measure students' understanding of mathematical concepts. The underlying philosophy that has driven many of these studies is that developing a conceptual understanding of mathematics is superior to memorizing mathematical facts and rules. Attempts to investigate this approach have taken various forms in recent decades. The term number sense was first utilized in the literature by Dantzig (1954), who noted:

> Man, even in the lower stages of development, possesses a faculty which, for want of a better name, I shall call Number Sense. This faculty permits him to recognize that something has changed in a small collection when, without his direct knowledge, an object has been removed or added to the collection. (p. 1)

However, the construct of number sense did not truly begin to be developed and researched until the National Council of Teachers of Mathematics (NCTM; 1989) first introduced it as an objective in their *Curriculum and Evaluation Standards for School Mathematics*, the development of which was in response to evolving societal and employment needs during the 1980s. The term number sense was included as a primary standard to be taught in Grades K–4 and a subcomponent of a standard for Grades 5–8. The term was first defined by the NCTM (1989) as "an intuition about numbers that is drawn from all the varied meanings of number" (p. 39), and its five components were listed as:

1. Developing number meanings
2. Exploring number relationships with manipulatives
3. Understanding the relative magnitudes about numbers
4. Developing intuitions about the relative effect of operating on numbers
5. Developing referents for measures of common objects and situations in their environment. (pp. 39–40)

Several of the foundational theories and studies that preceded and followed the publication of those standards are critical to include in this review to properly frame the recent approaches to studying number sense.

Baroody (1985) provided an early glimpse into the foundation of the number sense construct through the context of children learning basic number combinations. These include the 121 sums of the integers 0–10 as well as the corresponding subtraction, multiplication, and division combinations. Baroody suggested that while many viewed the knowledge of these combinations as *reproductive*, a process that emphasizes recall only, the process may at first be *reconstructive*. That is, early learners may naturally develop their own non-traditional strategies for arriving at mathematical conclusions. This suggestion was in contrast to prior foundational theories that claimed inventing individualized mathematical approaches was an attempt to avoid the work of memorization and was viewed as an obstacle to learning the number facts (Smith, 1921; Wheeler, 1939, as cited in Baroody, 1985). Baroody's work was formative in the advancement of future number sense frameworks, as number sense is rooted in the philosophy that flexible problem-solving strategies are beneficial to the development of mathematical understanding.

Greeno (1991) offered an early theory to frame the construct of number sense, describing it as "situated knowing in a conceptual domain" (p. 170). While others at the time viewed number sense through the lens of such measurable outcomes as flexible numerical computation, numerical estimation, and quantitative judgment and inference, Greeno (1991) proposed an interpretation of number sense as a conceptual environment:

> Numbers and quantities are important objects in the domain with a structure of relations and operations. People with number sense know where they are in the environment, which things are nearby, which things are easy to reach from where they are, and how routes can be combined flexibly to reach other places efficiently. They also know how to transform the things in the environment to form other things by combinations, separations, and other operations. (p. 185)

Greeno's theory proposed a different view on how learning mathematics is constructed, suggesting that number sense describes an environment that a person's mind can enter, as opposed to teaching in a way that puts number sense into their mind. Thus, Greeno suggested number sense could be considered global understanding, resulting from the exposure to and mastery of the full range of mathematical educational experiences.

McIntosh et al. (1992) published the first complete number sense framework, and their first attempt at defining number sense was to describe it as “a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations” (p. 3). Like Greeno (1991), McIntosh et al. (1992) suggested that number sense should be developed throughout students’ K–12 mathematics experiences, noting that number sense “is required by all adults regardless of their occupation and whose acquisition by all students should be a major goal of compulsory education” (p. 3). This reference highlights the importance of exploring adults’ number sense, which is the focus of this dissertation. McIntosh et al. (1992) supported Baroody's (1985) thesis by suggesting that the development of number sense strategies is a gradual process that develops over time, an indication that number sense can be learned, and any attempt to measure number sense will only indicate the subject’s level at that particular point in time. The formal framework presented by McIntosh et al. (1992) included three distinct components: “(1) knowledge of and facility with NUMBERS… (2) knowledge of and facility with OPERATIONS… (3) applying knowledge of and facility with numbers and operations to COMPUTATIONAL SETTINGS” (p. 4). Each of the components in this framework consisted of either three or four levels of understanding, and each level of understanding was divided into several subcomponents.

The use of flexible mathematical strategies is a hallmark of number sense, so some researchers examined students’ capacity to deconstruct numbers and use the parts to complete a mathematical process. Gray and Tall's (1994) work used studies that distinguished between more-able and less-able students, ages 7–12 years, to examine their approaches to solving simple arithmetic problems. Their hypothesis was that a significant indicator of mathematical ability was a student’s ability to seamlessly transition between concepts of mathematical *objects* and the *processes* used to manipulate those objects (Gray & Tall, 1994). For example, the + symbol indicates both the process of addition (such as finding a strategy to add 3 + 2) and the concept of sum, which is the result of addition (3 + 2 = 5). Gray and Tall observed that the more-able students did not necessarily rely on a greater set of known, or reproduced, facts, but they instead used those known facts to efficiently derive, or reconstruct, new facts. These findings aligned with Baroody's (1985) claim that reconstruction may be superior to reproduction in the development of mathematical understanding. Gray and Tall (1994), however, extended Baroody’s thesis by suggesting this ability in the more-able students led to a “built-in feedback loop which acts as an autonomous knowledge generator” (p. 132). Thus, Gray and Tall concluded that students who can seamlessly transition between different numerical and symbolic representations are able to efficiently manipulate prior knowledge to derive new knowledge. In contrast, some students are only able to stack complex processes, further increasing the difficulty of learning mathematics as they encounter additional challenges (Gray & Tall, 1994).

Gersten and Chard (1999) examined number sense through the perspective of special education and related the concept of number sense to *phonemic awareness*, which the authors describe as “the insight that words are composed of sounds” (p. 19). At the time of Gersten and Chard’s publication, number sense was often viewed as one’s ability to work flexibly with numbers through deconstruction and reconstruction (Baroody, 1985; Gray & Tall, 1994; Greeno, 1991; McIntosh et al., 1992). Gersten and Chard (1999) hypothesized that this is analogous to the development of reading strategies, noting, “Students are helped even more if they are provided with instruction not only in how to blend phonemes together, but also in how to ‘pull apart’ or segment words into phonemes” (p. 19). Their claim was that strategies similar to those used to help improve students' reading levels would also be beneficial to students struggling to develop number sense (Gersten & Chard, 1999). While the current study does not specifically target students with documented disabilities, the general population of students enrolled in community college developmental mathematics courses can be described as having mathematical abilities less than many of their peers. Thus, the work of Gersten and Chard (1999) is foundational to this study as it frames number sense "as a lens to reveal reasons for relative successes and failures of past attempts at innovations" (p. 18).

Berch (2005) advanced the work of many prior studies (Dehaene, 1997; Gersten & Chard, 1999; Markovits & Sowder, 1994; McIntosh et al., 1992, 1997; B. J. Reys, 1994; R. Reys et al., 1999; Sowder, 1992; Yang et al., 2004; Zanzali & Ghazali, 1999) in attempting to bring clarity to and define number sense. Berch observed that researchers had been inconsistently defining the construct of number sense and compiled a list of 30 different features of number sense that had been identified to that point in the literature. Specifically, Berch (2005) noted that number sense had been defined differently by cognitive scientists and mathematics educators and concluded:

> Possessing number sense ostensibly permits one to achieve everything from understanding the meaning of numbers to developing strategies for solving complex math problems; from making simple magnitude comparisons to inventing procedures for conducting numerical operations; and from recognizing gross numerical errors to using quantitative methods for communicating, processing, and interpreting information. (p. 334)

Berch's work helped clarify the construct of number sense to that point and has been referenced in many subsequent number sense studies.

Adjacent to number sense, yet crucial for thoroughly developing the theoretical framework of this study, is the construct of quantitative literacy. Maclellan (2012) conducted a qualitative analysis of the existing literature to explore the relationship between number sense and early quantitative literacy. Number sense, as described by Maclellan, involves number knowledge, counting skills and principles, nonverbal calculations, number combinations, and solving story problems. Quantitative literacy is a broader construct than number sense and, according to Maclellan, refers to one's ability to apply quantitative knowledge in the appropriate situation, although what differentiates *early* quantitative literacy from general quantitative literacy is not discussed. Maclellan concluded that the components of number sense are interdependent and the development of one's quantitative literacy relies upon having a strong foundation of number sense.

Maghfirah and Mahmudi (2018) conducted a thorough review of the existing number sense literature with attention given to its impact on student achievement. Maghfirah and Mahmudi (2018) used a narrower definition of number sense than many other researchers, succinctly describing it as "the ability to execute mental calculation without using any standard algorithm" (p. 1). Maghfirah and Mahmudi observed that number sense is generally predictive of students' academic success in mathematics and should be intentionally developed starting at a young age. Additionally, Maghfirah and Mahmudi highlighted the importance of environment, noting that the development of number sense is highly dependent upon one's learning experiences and requires a greater emphasis on numerical exploration than algorithmic calculations.

Whitacre et al. (2020) completed the most recent and most thorough examination of the existing number sense literature to determine how number sense has been used as a construct. The authors conducted a systematic review of 141 articles published through 2017 with number sense in the title and concluded that there are three distinct number sense constructs within the literature: approximate number sense, early number sense, and mature number sense (I. Whitacre et al., 2020). Whitacre et al. (2020) noted that these three constructs have different characteristics regarding seminal works, views on the origin and nature of number sense, key concepts, populations studied, and measurement instruments used. Whitacre et al. (2020) also observed that authors studying one number sense construct often cite studies from a different number sense construct in support of their findings, which raises concerns regarding validity. There are times when cross-construct analyses or references are reasonable, but these have often been done incorrectly and possibly without realization by many authors (Whitacre et al., 2020). Whitacre et al.'s (2020) description of mature number sense is used as a basis for the theoretical framework of the current study as it aligns most closely with the population being studied and the purpose of the research.

The current study, including the remainder of the literature review, will not attend to the concept of number sense as explored by the cognitive psychology community, which is founded on the theory that both humans and animals are born with an innate sense of quantity (Dehaene, 1997, 2011). With respect to Whitacre et al.'s (2020) review, this excludes research directed solely at the study of approximate number sense. Instead, the current study focuses on number sense as researched by those in the field of mathematics education. However, Whitacre et al. (2020) differentiated the mathematics education research into early number sense and mature number sense, with early number sense generally studying students in the grade ranges of preschool through second grade and mature number sense focusing on the students in the middle and secondary grades as well as preservice teachers. However, because number sense is learned (McIntosh et al., 1992), even mature number sense can begin developing in the primary school years and thus, analyses of the number sense of students in primary grades must be discussed.

Finally, Ghazali et al. (2021) conducted a systematic review of the definitions of number sense for students of ages 7–12 years. Their meta-analysis of the literature revealed five common themes from 48 categories: number composition, number identification, magnitude of number, number operation, and judgment making (Ghazali et al., 2021). Ghazali et al. (2021) used these themes to develop an inductive definition of number sense they described as "analogous numerical cognition perceived via logical linkages and thinking skills through various communication modes" (p. 7). Additionally, Ghazali et al.'s review confirmed the prevailing theme in the literature that number sense is a skill that is acquired and facilitated through educational mathematics experiences.

Number Sense Studies of Children and Adolescents

Most research on number sense has been focused on students at the K–12 level or the equivalent in international studies. This section of the literature review highlights studies that have investigated the number sense of children and adolescents participating in their compulsory education, and it is divided into three sections. The first section reviews students in the primary grades who are approximately 7–10 years of age. This is followed by a section concerning students in the middle grades who are primarily 11–13 years of age. The third section is focused on those at the secondary level who are generally 14–16 years of age. Some studies were conducted across multiple grade levels, and they are discussed in each of the appropriate sections.

Studies of Students in Primary Grades

In response to a growing focus on number sense in the field of mathematics education, particularly with the inclusion of number sense in the NCTM (1989) standards, McIntosh et al. (1997) conducted an international study of 1,712 students in Australia, Sweden, the United

States, and Taiwan. The impetus of this study was McIntosh et al.'s (1997) recognition that because the concept of number sense was in its infancy, most teachers lacked a true understanding of its meaning and therefore, were unsure how to assess it. Using the framework created by McIntosh et al. (1992), the researchers constructed a bank of questions designed to assess students' number sense for use by mathematics educators and researchers alike. The items were coded for use with appropriate age and grade levels. This instrument became the foundational item bank for the majority of future number sense research as many items were adapted to different studies' languages, ages, and cultures. This includes the assessment used in the current study, which is based on work by Whitacre (2012). McIntosh et al.'s (1997) research was conducted with students ages 8, 10, 12, and 14, generally coinciding with Grades 3, 5, 7, and 9, respectively. Those in Grades 7 and 9 will be discussed later in the literature review. In Australia and the United States, a mental computation test was conducted along with the number sense assessment, and the results suggested that as students grew older the correlation between mental computation and number sense grew stronger.

In 1999, R. Reys et al. conducted additional analyses on the results of McIntosh et al.'s (1997) work and concluded that the development of number sense does not necessarily coincide with the acquisition of computational skills, suggesting that emphasis on algorithmic procedures is not an effective teaching strategy for developing number sense. Additionally, R. Reys et al. (1999) noted that performance varied both within and across the different countries, although that performance was consistently low. However, no specific expectation was stated for what constituted high or low performance.

Yang (2003) conducted one of many international number sense studies with 67 Taiwanese fifth-grade students. Yang (2003) used the Number Sense Rating Scale, an instrument developed by Hsu et al. (2001, as cited in Whitacre, 2012). While most number sense studies were based on a single snapshot in time, Yang's (2003) was experimental and involved an intervention lasting four months. Number sense was measured in three phases—a pretest, a posttest, and a retention test five months following the intervention. Interviews were also conducted with certain students based on their pretest performance. Yang (2003) found that the students in the experimental class were more adept at using number sense strategies following the intervention and also performed better on the retention interviews than the control group. Yang (2003) was one of the first to conclude that number sense can be developed through appropriate instructional methods.

Menon (2004) conducted a study of 750 children in the United States in Grades 4–7 using a 10-question paper and pencil test with items drawn from R. Reys et al. (1999). Menon's study examined the number sense components of making mathematical judgments, developing useful and efficient strategies for managing numerical situations, and possessing a general understanding of numbers and operations. Follow-up interviews were conducted for students to clarify answers. For students in the primary grades, Menon found that for almost every item, the fourth-grade students performed better than the fifth-grade students, and the interviews confirmed that the older students were more likely to resort to learned procedures than rely on number sense.

In 2008, Yang et al. studied 1,212 Taiwanese students at the end of fifth grade. Yang et al. (2008) used a 23-item computerized number sense scale, a multiple-choice test originally developed for third-grade students, which also incorporated students' reasons for making their answer choices and allowed for no writing utensils. The average correct response rate was 46%, which Yang et al. (2008) classified as poor number sense, although they provided no basis for this other than their own expectations and experience. Additionally, Yang et al. (2008) analyzed the correlation between the students' number sense and their average end-of-year mathematics grades and found a significant correlation between number sense and mathematics achievement. In a follow-up study, Li and Yang (2010) used the same data set as Yang et al. (2008) to further assess the model and the data collection instrument. According to Li and Yang, most prior number sense studies had used five components as a basis for their instruments similar to the following: recognizing relative number size, using multiple forms of representations of numbers and operations, judging the reasonableness of computational results, understanding the basic meanings of a number, and using benchmarks to compare relative number size. Through exploratory factor analysis, Li and Yang concluded that for fifth-grade students, a better model would omit questions related to the factor of using benchmarks to compare relative number size. However, Li and Yang noted that while the four-factor model worked well as a quick number sense check for fifth-grade students, the factors that should be used depend upon what aspects of number sense the researcher finds most important.

Yang and Wu (2010) studied 60 Taiwanese third-grade students using four number sense components: understanding the basic meaning of numbers and operations, recognizing the relative and absolute magnitude of numbers, being able to use a benchmark appropriately, and judging the reasonableness of results. Like Yang (2003), Yang and Wu (2010) conducted an experimental study with a number sense intervention. They used a mixed-method analysis of 20 multiple-choice questions plus interviews and administered both parts three times: once as a pretest and twice as a posttest following two separate units of instruction. Yang and Wu (2010) used the interviews to categorize responses, both correct and incorrect, as number-sense-based, rule-based, or the wrong explanation. The experimental group was taught using pedagogical approaches that promote number sense, while the control group experienced standard activities from the textbook. Yang and Wu (2010) found that the experimental group performed significantly better on both posttests than the control group, suggesting that a number-sense-based teaching approach can help improve students' number sense and encourage them to solve problems in flexible ways without strictly relying on standard algorithms, confirming the findings of Yang (2003).

Mohamed and Johnny (2010) conducted a study in Malaysia with 32 high-achieving fourth-grade students. The researchers used a 20-item, multiple-choice number sense test developed by McIntosh (1997) and adapted by Zanzali and Ghazali (1999). These high-achieving students demonstrated an average correct response rate of 58% and were weakest in the components of recognizing the relative effect of operations and judging the reasonableness of computational results. Mohamed and Johnny (2010) also found a significant correlation between number sense and mathematics achievement, similar to the findings of Yang (2003) and Yang et al. (2008).

Mohamed and Johnny (2011) conducted a similar study to Mohamed and Johnny (2010), using the same test and assessing 261 high-achieving fourth-grade Malaysian students. However, Mohamed and Johnny (2011) conducted follow-up interviews with a selection of students to understand the strategies and reasonings behind their answers and also explored the difference between rural and urban students. They found a statistically significant difference in the results of urban and rural students, with the former averaging 56% correct and the latter averaging 43%. Mohamed and Johnny (2011) found that overall, students struggled to solve problems that were different from what they typically experienced in the classroom. Additionally, they experienced difficulty estimating without the use of written calculations, similar to the findings of Yang (2003) and Yang and Wu (2010).

Extending the work of Yang et al. (2008), Yang and Lin (2015) assessed the number sense of 195 Taiwanese fifth-grade students. Yang and Lin (2015) created a four-tier, 40-item, web-based test, which required students to include their confidence in each answer, a reason for their answer, and their confidence in the reason. Additionally, students were specifically encouraged to think flexibly and try to use non-traditional strategies. Overall, the researchers found students' correct response rate averaged 48% and they were much more confident in their answers than in their reasons. Furthermore, only 20% applied what Yang and Lin (2015) classified as number-sense-based reasoning to justify their answers.

Akkaya (2016) conducted a study in Turkey with 576 students in Grades 5–8 using a 50-question, multiple-choice test developed by Singh (2009). For the fifth-grade students, the average correct response rate was 18%, which Akkaya deemed very poor. The component in which the fifth-grade students performed most poorly was using multiple representations, similar to the findings of Yang and Lin (2015). There was a significant difference found between fifth-grade scores and those of the other grade levels, suggesting that number sense improved as they progressed into and through their middle grade mathematics education. This finding contradicted that of Menon (2004), who saw a decrease in the use of number sense strategies as age increased. Akkaya also examined the difference in number sense by gender and found none.

Yang (2019) and Yang and Sianturi (2019) conducted similar studies, the former with 819 fifth-grade students in Taiwan and the latter with 125 fifth-grade students in Hong Kong. Both studies used an online, three-tier test with 40 questions. The tiers explored students' knowledge and reasoning as well as their confidence levels in both. Yang (2019) observed a 45% correct response rate with the lowest number sense component being judging the reasonableness of a result, similar to Mohamed and Johnny (2010). Additionally, Yang (2019) found that students were generally overconfident and had significant misconceptions. With the Hong Kong students, who generally place at or near the top in international mathematics assessments, Yang and Sianturi (2019) observed a 68% correct response rate, which was significantly higher than that of the Taiwanese students, although just 34% demonstrated number sense in their reasoning. The researchers classified these results as unsatisfactory.

Studies of Students in Middle Grades

In an early foundational number sense study, Markovits and Sowder (1994) conducted an intervention with 10 seventh-grade students at a private school in the United States. The students received instruction that was designed to improve their ability to recognize number magnitude, perform mental computations, and perform computational estimation, all common indicators of number sense. The students were interviewed and completed three paper-and-pencil tests: a pretest before the instruction, a posttest following the instruction, and a retention test six months later. The results, which were confirmed in the retention test, suggested that the students had a better intuitive sense of number, transitioned away from standard algorithms toward more number-sense-based strategies, and were able to better use compatible numbers in estimation (Markovits & Sowder, 1994). Thus, Markovits and Sowder concluded that number sense can be taught in the middle grades when existing knowledge and intuition are leveraged to help students form new conceptual understandings.

In another foundational study, also discussed in the section on primary grades, McIntosh et al. (1997) examined the number sense of middle grade students in the United States, Australia, and Taiwan using items developed by McIntosh et al. (1992). In Taiwan, they also gave these students a 20-question, open-ended written computation test in which the questions mirrored those on the number sense assessment. McIntosh et al. (1997) found the Taiwanese students' number sense was significantly lower than their ability to perform written computations. As previously noted, a mental computation test was also administered in Australia and the United States. McIntosh et al. (1997) observed that as these students progressed from the primary grades to the middle grades, the correlation between their mental computational abilities and number sense grew stronger. R. Reys et al. (1999) reported on the results of the McIntosh et al. (1997) study, and, just as they did with the primary grade students, observed that the development of number sense in the middle grades does not necessarily coincide with the acquisition of computational skills. Thus, R. Reys et al. (1999) concluded that an emphasis on teaching algorithmic procedures does not promote growth in number sense.

R. E. Reys and Yang (1998) conducted a study in Taiwan with 115 sixth- and 119 eighth-grade students. Students were given a written computation test with 20 open-ended questions representative of national curriculum standards. They also completed a 40-item number sense test, the first half of which paralleled the questions on the written computation test. Seventeen students, nine of whom scored high on the tests and eight with scores in the middle, participated in follow-up interviews. Similar to the findings of McIntosh et al. (1997), students in both grades performed much better on the written computation test than the number sense test. It was observed in the interviews that the higher-level middle grade students were more likely to try number-sense-based approaches, but only when prompted to consider other strategies.

Yang and Huang (2004) continued the number sense research in Taiwan in a study with 627 sixth-grade students from 10 different schools. Four different 16-item tests were given, each with parallel questions, to assess students' performance in written computation, pictorial representation, symbolic representation, and number sense. Students scored highest in written computation and lowest in number sense. In comparing responses to parallel items on the tests, the researchers observed that correct answers do not always indicate correct thinking. Additionally, as noted in other studies (McIntosh et al., 1997; R. E. Reys & Yang, 1998), there was a disconnect between students' written computational ability and their ability to apply those skills flexibly to nonroutine contexts in which number sense could be used.

Yang et al. (2004) conducted another experimental study in Taiwan with 140 sixth-grade students from two schools. They assessed students' number sense using a paper-and-pencil test with 30 multiple-choice items. The assessment was given as a pretest, a posttest, and as a retention test. Additionally, students completed pre-, post-, and retention interviews, during which their responses were coded as either number-sense-based, rule-based, or could not explain. Although all students received instruction in the same standard curriculum, a control group's instruction focused on memorizing rules and performing written computation while an experimental group received number-sense-based instruction that encouraged them to explore mathematical concepts and discuss their thinking. Yang et al. (2004) found a statistically significant difference between the control and experimental groups' posttest and retention test scores, supporting other studies (Markovits & Sowder, 1994; Yang, 2003) that suggested number sense can be developed through intentional teaching strategies.

In a study previously discussed in the section on primary grade students, Menon (2004) assessed the number sense of 750 children in the United States in Grades 4–7 using a 10-question paper-and-pencil test with items drawn from the work of R. Reys et al. (1999). The rate of correct responses ranged from 19% to 52%, suggesting students possessed a low level of number sense, although Menon provided no indication as to what would be an acceptable level of number sense. Menon also observed that students in middle grades used inefficient strategies and tended to be more reliant upon procedures and algorithms when compared to those in the primary grades. Facun and Nool (2012) extended the work of Menon with sixth-grade students in the Philippines and found similar results. Using the same test as Menon, Facun and Nool also provided students with space to describe their reasoning on each item and examined students' performance in each number sense component. With correct responses ranging from 1% to 28% in each component, Facun and Nool concluded that the students generally lacked acceptable number sense.

Yang and Tsai (2010) explored how the use of technological tools could support the learning of number-sense-based strategies with sixth-grade students in Taiwan. In a quasi-experimental study with 32 students, the researchers administered a two-tiered, 50-question pretest and posttest that required students to provide both an answer and a reason. Students were scored on their answers and whether their reasoning was based on a procedural method or a number-sense-based method. Students were divided into a control group and an experimental group and taught the same lessons using strategies designed to enhance number sense. For additional support, the experimental group completed supplemental computer-based activities. Yang and Tsai's (2010) results indicated a statistically significant improvement in the experimental group's use of number-sense-based strategies compared to that of the control group, suggesting that integrating appropriate technological teaching tools can assist in the development of students' number sense.

In a study conducted in Turkey, Şengül and Gülbağcı (2012) assessed the decimal number sense of 573 students from six different schools in Grades 6–8. Each student completed a 16-question test and the researchers interviewed three students from each grade. Similar to

Mohamed and Johnny (2010), Şengül and Gülbağcı examined the results by component and found the lowest, with students averaging a 26% correct response rate, was judging the reasonableness of results. Unlike Menon (2004), Şengül and Gülbağcı found that middle grade students' number sense relating to decimals improved with each successive grade level, although even the eighth-grade students failed to score correctly on at least half of the test. As found in other studies (Menon, 2004; R. E. Reys & Yang, 1998; Yang & Huang, 2004), students preferred rule-based strategies as opposed to number-sense-based strategies, although in Turkey, at the time the study was conducted, number sense was not part of the national curriculum (Şengül & Gülbağcı, 2012).

Purnomo et al. (2014), in a study of sixth-grade students from three different Indonesian schools, used a 30-question test adapted from McIntosh et al. (1992) to assess three number sense components: knowledge of and facility with numbers, knowledge of and facility with operations, and knowledge of and facility with numbers and operations to computational settings. The students scored lowest, with a correct response rate of less than 27%, on knowledge of and facility with numbers. In alignment with other research (Menon, 2004; R. E. Reys & Yang, 1998; Şengül & Gülbağcı, 2012; Yang & Huang, 2004), the results of this study suggested that the students were much more fluent in procedural computations than number sense (Purnomo et al., 2014).

A study by Akkaya (2016), which was also referenced in the section on primary grade students, was conducted in Turkey with 576 students in Grades 5–8 using a 50-question, multiple-choice test developed by Singh (2009). When examining the results of all middle grade students, Akkaya found a statistically significant improvement in number sense between all grade levels except between sixth and seventh, although the overall correct response rate for all students, at just under 24%, was considered very poor. This general improvement in number sense in the middle grades was similar to the findings of Şengül and Gülbağcı (2012).

With 138 Turkish students in Grades 6–8, Gürefe et al. (2017) used a 17-question number sense scale to study three components of number sense: flexibility in calculation, conceptual thinking in fractions, and using benchmarks. Students' answers were judged and scored as either using number sense or using standard algorithms. Gürefe et al. found the overall scores to be very low, with the overall average for correctly using number sense at just 13%. There was a statistically significant difference with the eighth-grade students scoring better than the sixth- and seventh-grade students, in alignment with the findings of Akkaya (2016). Gürefe et al. also found no significant difference in number sense by gender, a similar finding to Yang et al. (2008).

Ulusoy (2020) used the same number sense scale as Gürefe et al. (2017) but with 70 sixth-grade students in Turkey. Employing an experimental approach, Ulusoy separated students into control and experimental groups with the control group receiving standard textbook-based instruction and the experimental group receiving instruction designed to enhance their number sense. Analyzing pretest and posttest data, Ulusoy found that the experimental group's number sense scores demonstrated statistically significant improvement, confirming the results of other studies (Markovits & Sowder, 1994; Yang et al., 2004; Yang & Tsai, 2010) that suggested number sense can improve with targeted instruction. However, unlike several other studies (Bütüner, 2018; Mohamed & Johnny, 2010; Yang et al., 2008), Ulusoy found no statistical difference between the mathematics achievement of the control and experimental groups.

Bütüner (2018) conducted a study to compare students' number sense to their previous mathematics achievement. Bütüner assessed 129 seventh-grade students from three elite schools in Turkey with 10 open-ended questions. Similar to the work of Gürefe et al. (2017), Bütüner coded students' answers as number-sense-based or rule-based. Additionally, students were ranked as low, medium, and high based on their mathematics course scores and assignment by teachers. Several from each group were interviewed. In alignment with other studies (Menon, 2004; Purnomo et al., 2014; R. E. Reys & Yang, 1998; Şengül & Gülbağcı, 2012; Yang & Huang, 2004), Bütüner found that most students used rule-based approaches on the test and hardly any used number sense strategies when interviewed. However, when number sense strategies were employed, the accuracy of their responses improved. While few questions were solved using number sense, the interviews revealed that the students with higher mathematics achievement tended to use number sense strategies more than those with low mathematics achievement, similar to the findings of Mohamed and Johnny (2010) and Yang et al. (2008).

Yang and Sianturi (2021) used the same framework and approach as Yang (2019) and Yang and Sianturi (2019), two studies that were discussed in the section on primary grade students. Yang and Sianturi (2021) assessed 308 Indonesian sixth-grade students with an online, three-tier, 40-question test. The three tiers examined students' knowledge, reasoning, and confidence levels in both knowledge and reasoning. The students' average correct response rate was 47%, but based on the researchers' classification system, only 25% demonstrated high or medium number sense and high confidence, suggesting an overall poor level of number sense (Yang & Sianturi, 2021). Students exhibited strong misconceptions on 27 of the 40 questions, and, similar to the findings of Şengül and Gülbağcı (2012), the component in which the students had the lowest correct response rate was judging the reasonableness of computational results, with an average correct response rate of just 27% (Yang & Sianturi, 2021).

Studies of Students in Secondary Grades

The foundational McIntosh et al. (1997) study, highlighted in the sections on primary and middle grade students, also included Grade 9 students in the research conducted in the United States, Australia, Sweden, and Taiwan. The secondary-level students were also assessed using items developed by McIntosh et al. (1992), although with a greater number of items. In Taiwan, they also gave these students a written computation test in which the questions mirrored those on the number sense assessment, while in Australia and the United States, a mental computation test was conducted as well. Similar to what was found in the younger grades, McIntosh et al. (1997) found the students' number sense was significantly lower than their ability to perform written computations, and as students transitioned into the secondary grades, their mental computational skills improved and the correlation between mental computational skills and number sense became stronger.

Following the McIntosh et al. (1997) study, few researchers explored the number sense of students in secondary grades until Singh (2009), who assessed 1,756 Malaysian students ages 13–16. The students were generally mathematically competent as approximately 75% scored an A on their end-of-year mathematics exams. Singh (2009) used a test adapted from McIntosh et al. (1997) with 50 multiple-choice questions. The results, disaggregated by grade level, ranged from 37% to 48%, with each successive grade level scoring slightly better than the previous, although there was no statistically significant difference between scores of the 13- and 14-year-olds and between the scores of the 15- and 16-year-olds (Singh, 2009). Singh (2009) found the results surprising, having expected scores to be higher and for students to develop significantly better number sense as they advanced from each grade level to the next.

Extending the research by Singh (2009), Aperapar and Hoon (2011) also conducted a mental computation test with the same students to analyze the association between number sense and mental computational ability. The results of the mental computation test ranged from an average of 79% to 89% between the different grade levels, considerably higher than the results of the number sense test. The researchers concluded that although mental computation is related to number sense, the students may have been trained to perform mental computational procedures at the expense of intuitive understanding. These results supported other studies that suggested reliance on standard written calculations may erode students' intuitive sense of number (Gersten & Chard, 1999; Givvin et al., 2011; McIntosh et al., 1992; R. Reys et al., 1999; Stigler et al., 2010; Tsao, 2004, 2005).

In a study of 215 Malaysian students in Grades 9 and 11, Singh et al. (2019) investigated if revised Malaysian standards for the primary grades had helped improve students' number sense. Using the same approach as Singh (2009), Singh et al. (2019) found that students' number sense scores remained low for the Grade 9 and Grade 11 students, at 45% and 50%, respectively. The scores were higher than they had been in the Singh (2009) study, although no statistical analysis was discussed in comparison between the two studies. Similar to Singh (2009), no significant difference was found between the younger students and older students, a conclusion Singh et al. (2019) determined was related to continued reliance on algorithmic teaching methods. Singh et al. (2019) did find a significant positive correlation between the students' number sense and their end-of-year mathematics scores for both grade levels, a similar finding to Bütüner (2018).

Number Sense Studies of College Students

There have been far fewer studies of college students' number sense than of those at the K–12 level. However, due to the increased recognition of the importance of number sense during these foundational educational years, there has been growing interest in the number sense of aspiring teachers who will ultimately be educating those students. Thus, most of the number sense research conducted at the post-secondary level has been completed with university students and not those at community colleges.

Studies of Undergraduate University Students

Hanson and Hogan (2000) conducted a study with 77 students enrolled in a university psychology course who had an average SAT score around the 70th percentile. Their study focused on estimation, which is an ability often utilized by those with strong number sense. Hanson and Hogan used three different versions of a 20-question estimation test that were administered in three different phases: once with a strict time limit, once with time for them to think aloud, and once with time to compute. What Hanson and Hogan found was that students were resistant to providing estimates, often wanting to demonstrate their ability to calculate exactly, and many were confused as to why an estimation was preferred over an exact solution. Hanson and Hogan also observed a strong desire by the subjects to revert to school-based algorithms and to follow mathematical rules, a common finding in many number-sense-related studies (Bütüner, 2018; Menon, 2004; Mohamed & Johnny, 2011; Yang, 2003; Yang et al., 2004; Yang & Wu, 2010).

Tsao (2004) conducted another early number sense study with college students. At a midsized university in the United States, Tsao (2004) studied 155 preservice elementary teachers enrolled in an entry-level mathematics course, comparing their number sense with their estimation ability, written computational ability, and mental computational ability. Number sense was assessed using 25 items from a test developed by Yang (1997), a number sense test that measured students' ability to recognize the relative magnitude of numbers, decompose and recompose numbers, use benchmarks, apply flexible mental computation and estimation strategies, and recognize the relative effect of operations. Unlike Aperapar and Hoon (2011), Tsao (2004) found a significant correlation between number sense and mental computational ability. Tsao (2004) also found number sense to be significantly correlated with written computational skills, a result that was contradictory to multiple prior studies (Aperapar & Hoon, 2011; McIntosh et al., 1997; R. E. Reys & Yang, 1998; Yang & Huang, 2004). Tsao (2004) concluded, in support of other research, that number sense does not necessarily develop in conjunction with written computational skills (Baroody, 1985; McIntosh et al., 1992; R. Reys et al., 1999).

Using the same 25-item number sense test as Tsao (2004), Tsao (2005) assessed the number sense of preservice elementary teacher students enrolled in an entry-level mathematics course at a midsized university in the United States. Based on the results, 12 students were selected to participate in interviews: six from the top 10% and six from the bottom 10%. What Tsao (2005) found was similar to the findings of other researchers: many students preferred to rely on procedures as opposed to number sense strategies (Bütüner, 2018; Menon, 2004). Like the preservice teachers studied by Hanson and Hogan (2000), Tsao (2005) observed that students were highly uncomfortable with estimation. Tsao (2005) also reported that some students in the high-ability group and most in the low-ability group demonstrated poor number sense, and their reluctance to apply number-sense-based strategies was rooted in a desire to use paper-pencil calculations. Tsao's (2005) findings supported other prior studies that suggested number sense is not cultivated in mathematical experiences that are grounded in standard computational procedures (Baroody, 1985; Gersten & Chard, 1999; McIntosh et al., 1992; R. Reys et al., 1999; Tsao, 2004).

Using a qualitative approach, Yang (2007) interviewed 15 preservice teachers in Taiwan: five majoring in mathematics education, five in elementary education, and five in language education. Yang (2007) observed that students were generally limited in their number sense. In alignment with other studies, Yang (2007) found that most students preferred using written, rule-based strategies (Bütüner, 2018; Menon, 2004; Tsao, 2004, 2005) and were more comfortable finding exact answers than with estimation (Hanson & Hogan, 2000; Tsao, 2005).

Yang et al. (2009) studied 280 preservice elementary teachers in Taiwan, most of whom were near the end of their university studies. The study investigated just two of the number sense components: using benchmarks in recognizing the magnitude of numbers and knowing the relative effects of an operation on various numbers. Students answered 12 questions, had to explain the reasoning for their responses, and were instructed to use estimation and mental calculations instead of written algorithms. Despite the instructions, the researchers observed that the majority of the participants completed written calculations to solve every problem, indicating that they did not know another way. Yang et al. (2009) deduced that children's lack of number sense may stem from their teachers' lack of number sense.

As a follow-up to Yang et al.'s (2009) results, Almeida et al. (2016) studied 67 preservice teachers with high mathematical ability enrolled in a mathematics degree at a university in Spain. In doing so, Almeida et al. were able to compare the number sense of preservice secondary teachers to the preservice elementary teachers from Yang et al.'s (2009) study. In addition to the 12-question test, Almeida et al. interviewed several of the participants to explore instances of many incorrect responses, unclear reasoning, and the use of a higher level of reasoning than was required. The results indicated that the preservice primary teachers scored slightly better than the preservice secondary teachers from Yang et al.'s (2009) research, but the reasoning used suggested significantly better number sense in the preservice secondary teachers.

I. M. Whitacre (2012) analyzed the number sense of prospective elementary teachers at a large urban university in the United States. I. M. Whitacre (2012) utilized a 37-question Number Sense Rating Scale, which was also adapted for use in the current study. It was given to 34 students enrolled in a course designed to improve their number sense as both a pretest and posttest. Subjects were also interviewed at the conclusion of the course. I. M. Whitacre's (2012) results suggested that, given specific instruction, the number sense of college students can improve. Using the same instrument and testing approach, I. Whitacre and Nickerson (2016) assessed first-year students enrolled in the first of four mathematics courses for prospective elementary teachers. The researchers' purpose for this study was focused primarily on the students' understanding of fraction magnitude rather than on other components of number sense. The course in which the students were enrolled included instruction designed to develop their number sense. I. Whitacre and Nickerson (2016) found a statistically significant increase in the mean score on the assessment from 65% to 79%. Additionally, I. Whitacre and Nickerson (2016) reported that the number of strategies students used to solve problems increased from 4.9 to 7.6—evidence of improved computational flexibility—and there was more balance observed between the use of standard and non-standard strategies.

At a state university in Turkey, Yaman (2015) assessed 74 third-grade teacher candidates who were in their third year of university study. The participants were given a 17-question number sense test before and after a sequence of two mathematics education courses designed to improve their number sense. The test measured three subscales of number sense: flexibility in calculation, conceptual thinking in fractions, and the use of reference points. Ten random participants were also interviewed before and after the courses. As found in other studies, the students generally preferred to use formulas and rules instead of number-sense-based strategies (Bütüner, 2018; Menon, 2004; Tsao, 2004, 2005; Yang, 2007; Yang et al., 2009). However, students' number sense showed statistically significant improvement, with the average score on the test increasing from approximately 41% to 53%, supporting other findings that suggest number sense can be enhanced with intentional instruction (Markovits & Sowder, 1994; I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016).

Studies of Developmental Mathematics Students

Other researchers have investigated the mathematical ability of college students enrolled in developmental mathematics courses, although few of those studies focused on the specific construct of number sense as generally defined by the literature. Papers by Stigler et al. (2010) and Givvin et al. (2011) represent a single study published in two parts about community college developmental mathematics students. Stigler et al. analyzed data from 5,830 custom placement tests given to students at a California community college to determine items related to algebra readiness that they found the most challenging. The authors also conducted a survey of 748 students enrolled in Los Angeles area community colleges, in which students were asked to explain their reasoning in answering several questions from the developmental mathematics curriculum (Stigler et al., 2010). The questions explored students' understanding of numbers and operations, including the ability to work flexibly with numbers. Although the researchers did not restrict their study specifically to number sense and did not reference number sense as defined by literature, Stigler et al. (2010) noted that many student errors occurred because "rather than using number sense, students rely on a memorized procedure, only to carry out the procedure incorrectly or inappropriately" (p. 9). Interestingly, Stigler et al. found very few differences in the reasoning ability of students who placed into a prealgebra course compared to those who placed into an elementary algebra course, suggesting that the higher placement of the latter could have simply been due to their ability to remember more procedures. Stigler et al. noted the fragility of this type of mathematical competence, just as Gray and Tall (1994) concluded that the less-able students tend to possess less mathematical flexibility and rely on stacking complex processes, which makes progressively more difficult mathematical problems even more challenging.

Givvin et al. (2011) conducted follow-up interviews with 30 of the developmental mathematics students from the Stigler et al. (2010) study who participated in the survey. The focus of their questioning was to uncover students' conceptual understanding and their ability and inclination to reason through problems. Like Stigler et al., Givvin et al. did not specifically investigate number sense as defined in the literature. However, the items used in the interviews by Givvin et al. were similar to those on the number sense assessment used in the current study, suggesting that they are conducive to assessing number sense, although several problems used by Givvin et al. were more algebraic in nature. Givvin et al.'s qualitative approach was to assess students' reasoning ability by analyzing how often they relied on procedural methods to solve problems when an intuitive method would have been much simpler (such as number decomposition while applying the distributive property). Givvin et al. considered the effect that school has on some students and concluded that emphasizing procedural approaches often results in students' inclination to default to procedures and a reluctance to reason through problems. Givvin et al. used the term *conceptual atrophy* to imply that students entering elementary school generally have an intuitive sense of quantity, but educational practices that emphasize procedural methods gradually erode that ability, a phenomenon that was also observed in studies by Facun and Nool (2012) and Menon (2004). These findings are similar to those who found preservice teachers enrolled in college-level mathematics coursework also often resorted to procedural approaches when a number sense strategy may have been more appropriate (Tsao, 2005; Yaman, 2015; Yang, 2007; Yang et al., 2009). Givvin et al. also observed that, while reasoning through problems was not most students' default approach, it could be drawn out of them with leading questions and clear permission to use nontraditional strategies, similar to the findings of R. E. Reys and Yang (1998).

Ali (2014) conducted a study of developmental mathematics students at a Pennsylvania university in which 29 students completed a 10-question number sense assessment, which required them to include their reasoning. Ali's questions were derived from standard number sense components such as using numbers flexibly, using benchmarks to make judgments, making reasonable estimates, and judging the reasonableness of answers. The results of Ali's study support the findings of other studies that suggest college students prefer algorithms and procedural approaches to using number sense and reasoning (Givvin et al., 2011; Stigler et al., 2010; Tsao, 2005; Yaman, 2015; Yang, 2007; Yang et al., 2009).

Steinke (2017) conducted the only study with community college developmental mathematics students specifically designed to assess their number sense. However, the researcher employed a unique approach to identify adults who lacked whole number sense skills. Unlike most previous number sense studies that defined number sense through several common components, Steinke (2017) contended that a student's number sense was based upon their internalization of two concepts, "1) the 'equal distance of 1' that exists between neighboring whole numbers . . . and 2) part-whole coexistence (the parts and whole exist at the same time)" (p. 5). Working under this framework, Steinke developed an assessment in which students plotted five numbers on a 0-to-20 number line. This assessment was given to 657 students enrolled in three levels of developmental mathematics courses, and the students were classified into three stages based on their performance. Steinke found that students who performed at the highest stage generally completed their courses with higher grades and had lower withdrawal rates than the students whose ability placed them into the lower two stages of number sense. Steinke's findings were similar to several studies designed for students in the primary grades, (Mohamed & Johnny, 2010; Yang et al., 2008), middle grades (Bütüner, 2018), and secondary grades (Singh et al., 2019).

Summary

As discussed in this review of the literature, community college students who are referred to developmental mathematics generally have poorer academic outcomes than those who begin immediately in college-level work (Burley et al., 2009; Cox & Dougherty, 2019; Xu & Dadgar, 2018). The reasons for the difference in these outcomes are complex and may be dependent upon instructional approaches (B. V. Cafarella, 2014; Cox, 2015), students' academic engagement (B. V. Cafarella, 2014; Pruett & Absher, 2015; Smith et al., 1996), and factors external to college (B. V. Cafarella, 2014; Fong et al., 2015). Colleges have implemented various strategies to help improve these outcomes, including mindset interventions (Center for Community College Student Engagement, 2019; Mills & Mills, 2018; Paunesku et al., 2015; Samuel & Warner, 2021), acceleration models (B. Cafarella, 2016; Hodara & Jaggars, 2014; Jaggars et al., 2014, 2015), modified placement policies (T. Bailey et al., 2010; Barnett et al., 2018; Kosiewicz & Ngo, 2020; Ngo et al., 2018; Scott-Clayton et al., 2014), corequisite remediation (Logue et al., 2019; Park et al., 2018; Ran & Lin, 2019), and complete redesigns of mathematics pathways (T. R. Bailey et al., 2015; Bishop et al., 2018; B. V. Cafarella, 2016; Rutschow et al., 2019). The most widely implemented of these changes have been to the systemic structure of developmental mathematics, and few have addressed the weaknesses that this population of students may have in their foundational mathematical skills.

Within the system of compulsory education, both in the United States and internationally, there has been a growing focus on students' number sense over the past 30 years. Although many of the studies included in this literature review were conducted internationally, and the different cultures and national curriculum standards may play a role in academic performance, number sense was consistently observed as unsatisfactory across borders (Akkaya, 2016; Facun & Nool, 2012; Gürefe et al., 2017; Menon, 2004; R. Reys et al., 1999; Singh et al., 2019; Tsao, 2005; Yang, 2007; Yang et al., 2009; Yang & Huang, 2004; Yang & Sianturi, 2019, 2021). A common theme found in the literature was that procedural-based approaches to teaching mathematics may hinder the development of number sense (Aperapar & Hoon, 2011; Givvin et al., 2011; Tsao, 2005). Conversely, number-sense-based instruction has been shown to be effective at developing the number sense of students at all levels (Markovits & Sowder, 1994; Ulusoy, 2020; I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016; Yaman, 2015; Yang, 2003; Yang et al., 2004; Yang & Tsai, 2010; Yang & Wu, 2010). Finally, students with greater number sense were generally found to have better mathematics achievement (Bütüner, 2018; Mohamed & Johnny, 2010; Singh et al., 2019; Steinke, 2017; Yang et al., 2008).

At the community college level, very little research has been conducted within the construct of number sense, a gap in the literature that the current study will begin to fill. The following chapter explains the methodology used in this study to better understand the number sense of community college developmental mathematics students, how their number sense may change by completing a developmental mathematics course, and how their number sense may be associated with their course outcomes.

CHAPTER 3: METHODOLOGY

Introduction

Students who place and enroll in developmental mathematics courses at a community college are less likely to persist toward completing a college degree than those who arrive ready for college-level coursework (Burley et al., 2009). Although this problem has received considerable attention in recent years from a multitude of organizations, many of the approaches to improving student outcomes have focused on changing institutional structures, few of which have had a significant impact on these students' retention and completion rates (Rutschow et al., 2022). The purpose of this study is to explore the problem by investigating community college developmental mathematics students' number sense and how it may be associated with academic success in their courses. This quantitative study was designed to answer the following research questions:

1. What level of number sense do community college developmental mathematics students have?
2. Is there a difference between the number sense of students enrolled in different developmental mathematics courses?
3. Does completing a traditional community college developmental mathematics course improve students' number sense?
4. Do students with greater number sense have better academic success in community college developmental mathematics courses?

This chapter explains the design of the study, the sampling procedure, how the data were collected and analyzed, assumptions that were made, the study's reliability and validity, and any ethical considerations.

Research Design

Philosophical Perspective

A correlational quantitative methodology was chosen to answer the research questions. Quantitative research tends to be founded on a philosophical perspective in which one assumes that an observable truth exists and that it can be measured (Merriam & Tisdell, 2016). Qualitative research, on the other hand, is rooted in the philosophy that no single reality exists and that understanding a phenomenon requires examining individual participants' perspectives (Tuli, 2010). Although a qualitative approach may have provided a better understanding of the nature of students' number sense and their mathematical thinking, the purpose of this study is rooted in the problem of poor academic success for students who place and enroll in developmental mathematics courses. Thus, a research approach that could identify a possible relationship between the students' number sense and their academic success is more appropriate. While many underlying factors may influence students' success in their developmental mathematics courses, the fact that they were identified upon their entrance to college as being underprepared for college-level mathematics suggests a deficiency in their foundational mathematical abilities. Traditional developmental mathematics courses are often designed to reteach many of the same concepts using the same methods that students were exposed to in their compulsory educational experiences. If a previously unidentified relationship could be established between students' number sense and their likelihood of academic success, this could suggest a reason for redesigning both the content of the traditional developmental mathematics curriculum as well as the teaching methods used in the courses. For these reasons, a correlational quantitative methodology is most appropriate for this study.

Although many studies on number sense have been conducted at the K–12 level and with preservice teachers in universities, the number sense of community college students has largely been ignored. Therefore, it was appropriate to limit the scope of this study and conduct research that provides descriptive data on community college developmental mathematics students' number sense and investigates possible correlations between number sense and academic success. The results of this study may help provide a foundation for future researchers to expand on the findings.

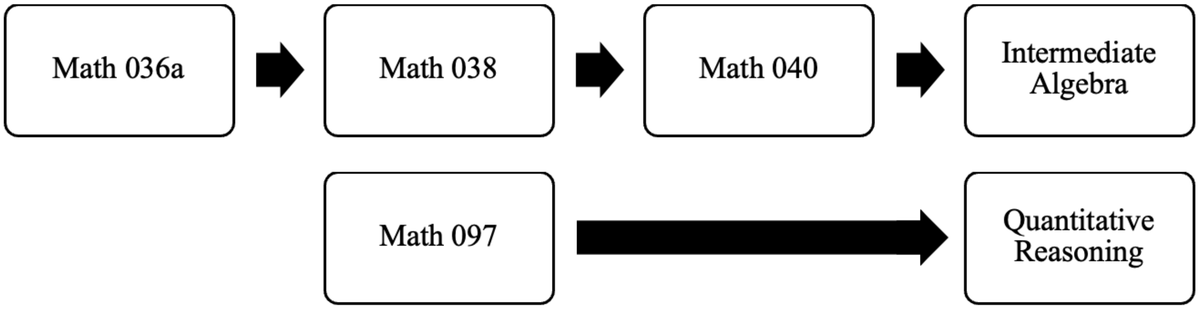
This study examines the number sense of community college students enrolled in four different developmental mathematics courses, and it seeks to determine if their number sense may improve throughout a typical developmental course and if there is a relationship between their number sense and success in their respective courses. Variations of a number sense assessment, originally created by Hsu et al. (2001, as cited in I. M. Whitacre, 2012), have been used to explore similar questions with other student populations (I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016; Yang, 2003). This study used a slightly modified version of this instrument as a pretest and posttest during the fall semester of 2021, along with students' final course grades, to attempt to answer the research questions.

Sampling Procedure

The population studied was developmental mathematics students enrolled at a community college. All students who were enrolled in a developmental mathematics course, held in person or synchronously online, in the fall of 2021 were invited to participate in the study. Thus, the entire population was included, other than one asynchronous online section. This asynchronous online section was excluded because the assessment needed to be administered in a synchronous environment, as described below. The participants were students who were at least 18 years of age at the time of the pretest and enrolled in one of the following four courses at Muskegon Community College in the fall of 2021: Math 036a (Basic Math), Math 038 (Prealgebra), Math 040 (Beginning Algebra), and Math 097 (Math Literacy). A total of 271 students began the semester in a synchronous section of one of those four courses, of which 165 completed the pretest and 100 completed the posttest.

The content taught in each of the mathematics courses was similar to the content of most traditional community college developmental mathematics courses, and a detailed list of objectives can be found in Appendix D. Historically, the traditional student progression through the course sequence would begin with Math 036a, followed by Math 038, and then Math 040, before moving on to an intermediate algebra course or a quantitative reasoning course, depending on the student's degree requirements. Math 097 was introduced in the fall of 2021 as a new, single-course developmental prerequisite for the quantitative reasoning pathway, as shown in Figure 1. Math 036a, Math 038, and Math 040 remained as prerequisite developmental courses in the STEM pathway leading to Intermediate Algebra. At the time of registration, students chose between enrolling in a course in the quantitative reasoning pathway or a course in the traditional STEM pathway. Students were provided guidance on the appropriate mathematics pathway through academic counseling and in the college catalog. Students choosing programs that required College Algebra or higher were directed to the STEM pathway, while students choosing programs that did not required College Algebra were directed to the quantitative reasoning pathway (Muskegon Community College, 2021a).

Figure 1: Developmental Mathematics Course Sequence by Degree Pathway



Additionally, due to COVID-19 restrictions, the college's capacity to administer placement tests was limited, and students were granted the ability to self-place in the fall semester of 2021, bypassing any prerequisites if they chose to do so. Thus, there was no mandated standard in place for assigning students to different levels of mathematics. However, students were provided guidance through academic counseling and on the college's website (Muskegon Community College, 2019, 2021b), as shown in Table 1.

Table 1: Developmental Mathematics Placement Guidelines for Fall 2021

Placement test	Math 036a: Basic math	Math 038: Prealgebra	Math 040: Beginning algebra	Math 097: Math literacy
SAT/PSAT math sub-score	13–19.5	20–22.5	23–25.5	13–25.5
ACT math	10–14	15–16	17–18	10–18
Accuplacer Nextgen QAS		200–230	231–260	200–260
Accuplacer Nextgen arithmetic	200–250	251–270	271–300	200–300

Data Collection

The data collected included students' number sense at the beginning of their mathematics course, their number sense at the conclusion of their mathematics course, and their final grades in their mathematics course during the fall semester of 2021. The students' number sense was measured using a number sense assessment, which was adapted from other researchers' work (I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016; Yang, 2003). Students' final course grades were acquired from the college's student records database. The following sections describe the data collection instrument and procedures in detail.

Instrumentation

The data collection instrument used was a number sense assessment that was originally developed by Hsu et al. (2001, as cited in I. M. Whitacre, 2012) and adapted for the purposes of this study. The instrument was originally developed for use with fifth- and sixth-grade students (Hsu et al., 2001, as cited in I. M. Whitacre, 2012), and other researchers have used it in similar studies to assess the number sense of various student populations, including K–12 students and preservice teachers in universities (I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016; Yang, 2003). The number sense assessment used in this study, found in Appendix A, contained 37 questions, and the same assessment was used as both a pretest and posttest. Just as other researchers (Li & Yang, 2010; Yang et al., 2008; Yang & Lin, 2015; Yang & Sianturi, 2019) have conducted studies using computerized or web-based versions of number sense tests, this number sense assessment was administered using the FlexiQuiz website (NextSpark Pty Ltd, 2021). The instrument itself is a multiple-choice assessment that asks mathematical questions designed to be answered using only mental mathematical reasoning with no pen, paper, or calculator. The questions originated from the work of McIntosh et al. (1997), who developed the foundational number sense framework and a corresponding bank of nonroutine questions designed "to elicit strategy generation based on understanding rather than strategy recall based on familiarity with problem-type" (pp. 8–9). To keep students from having the opportunity to complete standard procedural calculations, only one question was available at a time, each question had a time limit of 45 seconds, and no backtracking was permitted. At the conclusion of the 45 seconds, the testing software automatically advanced to the next question if the student had not yet done so themself. This time constraint was in alignment with other number sense studies using similar instruments (McIntosh et al., 1997; R. E. Reys & Yang, 1998; Tsao, 2004, 2005; I. Whitacre & Nickerson, 2016; Yang & Huang, 2004).

To ensure the instrument was appropriate for the population being studied, multiple college mathematics instructors reviewed the test items and provided feedback. Additionally, a pilot test was conducted with three different developmental mathematics courses at the same institution during the summer of 2021. A total of 40 students participated in the pilot test and each had the opportunity to provide feedback about the time constraint and the phrasing of the questions. The majority reported that the time limit was just about right or more than enough. Follow-up interviews were held for students to share any additional concerns. Based on the follow-up student interviews and the feedback from the instructors, several questions were rephrased for clarification.

Data Collection Procedures

This study had two data collection phases: one for a pretest and one for a posttest. The pretest phase occurred between August 23 and September 1 of 2021, during one of the first four meetings of each course section. The second phase took place during one of the final three meetings of each section prior to the final exam. Different course sections varied in their number of weeks. Thus, the second phase began as early as September 29 for one section and ended on December 1 for others. In the case of both phases, arrangements were made in advance with each instructor to determine the best date for the testing to occur in their respective classes.

Both phases were conducted using the same procedures, the same instrument, and the same administrator. The number sense assessments were administered by a single member of the department's office staff. For the classes that were held on campus, the test administrator led the students to a computer lab at the appropriate times. For the classes that were held synchronously online, the test administrator joined the video conferencing platform at the scheduled times. The synchronous online classes used either Zoom or Blackboard Collaborate, based on each instructor's preferred platform.

Using pretest and posttest scripts designed by the researcher, as shown in Appendix B and Appendix C, respectively, the test administrator introduced the purpose of the study and shared its potential benefits to the institution as well as the potential risks of participation. Students were provided with the appropriate information regarding consent so they could choose whether to have their results included. Although students were free to opt out of participation in the research study, they were all asked to complete the number sense assessment as part of the college's standard internal assessment activities.

Following the introduction of the study, the students were provided the web address with which to access the online number sense assessment on the FlexiQuiz website (NextSpark Pty Ltd, 2021). For the students participating in person, the web address was distributed on individual slips of paper that they typed into a web browser. For those participating synchronously online, it was pasted into the platform's chat feature, at which point students could click on the hyperlink to open the test. Once all students had accessed the website, the administrator read the instructions for the test, which included a 45-second time limit on each question, restricted access to any writing utensils or a calculator, and did not allow backtracking to previously submitted questions. Once students received the instructions, they confirmed or declined consent to their results being included in the study and then began. Upon completion of the test, the instructors resumed their regularly scheduled class activities. The entire process took approximately 30–40 minutes for each class.

At the conclusion of the 2021 fall semester, the results from the number sense pretest and posttest were downloaded from the FlexiQuiz website (NextSpark Pty Ltd, 2021) into a spreadsheet in Microsoft Excel (Version 2018). Additionally, final course grades were acquired from the college's student records database, Ellucian Colleague, using the Entrinsik Informer data analytics tool. The pretest scores, posttest scores, and final course grades were then combined into a single Excel spreadsheet, which served as the primary data file.

Data Analysis

The full data set consisted of students' pretest and posttest scores on the number sense assessment, the mathematics courses in which they were enrolled, and their final course grades on a standard 0–4.0 scale. If students did not complete either the pretest or posttest, their scores were left blank. For students who dropped the course or received an incomplete, their final course grades were left blank. These data were imported into IBM SPSS Grad Pack Premium v27 for analysis. This section describes the approaches used to analyze the data to address each of the research questions and test the stated hypotheses.

Research Question 1

This question asked: What level of number sense do community college developmental mathematics students have? The data used to answer this question included the pretest scores of the students who took the number sense assessment at the beginning of the semester and the mathematics courses in which they were enrolled. Students' number sense was measured by a 37-question multiple-choice number sense assessment. Correct answers were assigned a score of

1, while incorrect answers and those not answered were assigned a score of 0. Thus, the minimum possible score was 0 and the maximum possible score was 37. This research question was investigated using descriptive quantitative statistics, including mean and standard deviation.

Research Question 2

This question asked: Is there a difference between the number sense of students enrolled in different developmental mathematics courses? Using the same data set as Research Question 1, this question was analyzed using inferential statistics. The null hypothesis for Research Question 2 was:

> H_0: There is no significant difference between the number sense of students who are enrolled in different developmental mathematics courses.

This was tested against the following alternative hypothesis:

> H_a: There is a significant difference between the number sense of students who are enrolled in different developmental mathematics courses.

To examine this question, the independent variable was the mathematics course in which the students were enrolled, a nominal-level measurement. The dependent variable was the students' number sense at the beginning of the semester, which is an interval-level measurement. The data were analyzed using a one-way Analysis of Variance (ANOVA) test.

Research Question 3

This question asked: Does completing a traditional community college developmental mathematics course improve students' number sense? The data used to answer this question included the mathematics courses, the pretest scores, and the posttest scores of the students who took the pretest and the posttest. This question was explored using inferential statistics. The null hypothesis for Research Question 3 was:

> H_0: As a result of completing a traditional community college developmental mathematics course, there will be no significant increase in students' number sense.

This was tested against the following alternative hypothesis:

> H_a: As a result of completing a traditional community college developmental mathematics course, there will be a significant increase in students' number sense.

The independent variable for Research Question 3 was the completion of a traditional developmental mathematics course. Completion of a course indicates that a student did not withdraw from the course and received a final letter grade at the conclusion of the semester as measured on a standard 0–4.0 scale. The dependent variable was the change in students' number sense, which was measured at the beginning and end of the semester using the number sense assessment. Only students who completed the pretest and the posttest were included in this portion of the study. Results were analyzed for all students and were disaggregated by each of the four developmental mathematics courses. Paired samples *t* tests were used to determine if completing the courses resulted in improved number sense.

Research Question 4

This question asked: Do students with greater number sense have better academic success in community college developmental mathematics courses? The data used to answer this question included the mathematics courses, the pretest scores, and the final grades of the students who took the pretest at the beginning of the semester and earned a letter grade in the course. This question was explored using inferential statistics. The null hypothesis for Research Question 4 was:

> H_0: There is no significant correlation between students' number sense and their success in community college developmental mathematics courses.

This was tested against the following alternative hypothesis:

> H_a: A significant, positive correlation exists between students' number sense and their success in community college developmental mathematics courses.

For Research Question 4, the independent variable was students' number sense at the beginning of the semester, as measured by the number sense assessment. The dependent variable was the students' academic success in their developmental mathematics courses. Academic success was measured by the students' final grades in their developmental mathematics course on a standard 0–4.0 scale, which is an ordinal-level measurement. A non-parametric test, the Spearman rank-order correlation, was conducted to determine whether there was a significant relationship between number sense and final course grades.

Reliability, Validity, and Generalizability

For a study to be dependable, the results presented must represent the truth as accurately as possible. The level of dependability is based on the study's reliability and validity. Several approaches were utilized in the design of this study to help ensure the reliability of the measurement tool and the validity of the propositions.

Reliability

Reliability is primarily based on the instrument that is being used to measure a construct. The more consistently the instrument measures a construct, the more reliable the instrument is (Trochim et al., 2016). The instrument adopted in this study was designed by Hsu et al. (2001, as cited in I. M. Whitacre, 2012) who drew items from a bank of number sense questions created by McIntosh et al. (1997) in a foundational number sense study. Hsu et al.'s (2001, as cited in I. M. Whitacre, 2012) instrument has also been used in several other number sense studies involving various populations of students (I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016; Yang, 2003). Using this previously tested instrument helps ensure the reliability of measuring number sense in this study.

Additionally, reliability improves when testing conditions are consistent. To ensure consistency, the test in this study was administered to all students by the same person using the same script. Although some students were participating in person and others were participating synchronously online, all students completed the test on a computer. Thus, the instructions and procedures remained as consistent as possible and the students' interaction with the instrument remained identical.

The reliability of an instrument can also be estimated by its internal consistency, which indicates that individual questions are measuring the same construct (Maciocha, 2012). A common measurement of this internal consistency is the Chronbach alpha coefficient, which ranges from 0 to 1. According to Hooper (2012), the greater the Chronbach alpha coefficient, the more reliable the instrument, with .7 being generally accepted as a minimum. For the pilot test, Chronbach's alpha was calculated to be .86, indicating a good level of internal reliability. Minor changes were made following the pilot test to improve the clarity of several questions. Chronbach's alpha was recalculated for both the pretest and posttest and found to be .83 and .85, respectively. This suggests that the test maintained a good level of internal consistency and thus, reliability.

Validity

Validity, according to Trochim et al. (2016), is the degree to which the results accurately represent the truth. There are multiple types of validity, but because this is a correlational study, it is especially important to ensure *conclusion* validity (Trochim et al., 2016). Conclusion validity, which is "the degree to which conclusions you reach about relationships in your data are reasonable" (Trochim et al., 2016, p. 281), can be improved by increasing the statistical power of the study. For Research Questions 2, 3, and 4, each of which explores correlations within the data, the statistical power was calculated using IBM SPSS Grad Pack Premium v27 and found to be .933, .999, and .944, respectively. A larger statistical power, which ranges in value from 0 to 1, is better, and any statistical power larger than .8 is generally acceptable for social science research (Trochim et al., 2016). Thus, the results of this study can be accepted as having good conclusion validity.

Additionally, *construct* validity describes the extent to which the instrument measures what it is intended to measure (Trochim et al., 2016). As noted in the section on reliability, the instrument being adopted in this study has been used in multiple prior research studies (I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016; Yang, 2003). Using a previously tested instrument such as this helps improve the construct validity of the study (Trochim et al., 2016). Furthermore, several community college developmental mathematics instructors reviewed the instrument to confirm that the questions were appropriate for the population being analyzed. These approaches helped ensure that the instrument used in this study accurately measured number sense and improved the dependability of the results.

Delimitations, Limitations, and Assumptions

Despite the actions taken to ensure the reliability and validity of the research design, the results may not be able to be generalized to other institutions and other groups of students due to several factors, including the delimitations, limitations, and assumptions.

Delimitations. To narrow the scope of the research, the following delimitations were placed upon the design of this study:

1. The dependent variable for Research Question 4 was limited to fall 2021 mathematics course grades.
2. Data were collected from only one educational institution, a medium-sized community college in the midwestern United States.

3. Students' number sense was tested using a single quantitative instrument. Such multiple-choice tests can only examine students' abilities to identify correct answers within a given time constraint. While the questions were designed to capture students' capacity to utilize number sense strategies as opposed to traditional written calculations, an incorrect response does not imply they were not using number sense approaches (Yang & Lin, 2015).

4. Follow-up interviews were not conducted and could have revealed a deeper understanding of students' thinking.

5. Only students 18 years of age and older were allowed to participate.

6. Only students enrolled in synchronous classes were invited to participate, which excluded one section of 40 students enrolled in an asynchronous online Math 040 (Beginning Algebra) class.

7. Factors such as age, years since last mathematics course, gender, income level, instructor, and length of course were not considered.

8. Non-academic factors that can affect students' grades such as childcare, work schedules, motivation, adherence to academic integrity, or health were not considered.

Limitations. Additionally, the following unintended limitations occurred throughout the research process:

- Due to COVID-19 restrictions limiting the college's capacity to administer placement tests, students were allowed to self-place in the fall semester of 2021, bypassing any prerequisites if they chose to do so. Thus, there was no mandated standard in place for assigning students to different levels of mathematics. Students were provided guidance through academic counseling and on the college's website (Muskegon Community College, 2019, 2021b).

- For consistency, students participating on campus and students participating synchronously online were both provided the same online testing instrument. The instructions indicated that students should answer questions by thinking through the problems and not by using a calculator, a pencil and paper, or other aids in calculation. However, there is no way for the researcher to confirm that the students participating synchronously online did not use such resources.

- Although all students were asked to complete the pretest and posttest for use in the college's general assessment reports, students were given the opportunity to have their results excluded from this research study. There may have been factors that are correlated with number sense, such as mathematical confidence, that contributed to students' decisions to have their results included in the study.

- Some students were absent on the day that either the pretest or posttest was given, and some students dropped their course during the semester or received an incomplete. Specifically, out of the 271 students enrolled in a developmental mathematics course to begin the semester, 165 (61%) participated in the pretest phase of the study. Of those 165 students, 148 (90%) earned a final letter grade in their course and 100 (61%) participated in the posttest phase.

Assumptions. Finally, the following assumptions were necessary in the conduct of this research:

- Although students were able to self-place into their mathematics courses, they did so accurately.
- The instrument used in this study appropriately measures students' number sense.
- Students took the number sense assessment seriously and performed to the best of their abilities.
- Students did not use any writing utensils or calculators while answering the questions on the number sense assessment. The time limit set for individual questions was intended to restrict this opportunity.
- Students participating in a synchronous online class from a remote location performed similarly to those students participating in a traditional, face-to-face class on campus, both in their coursework as well as on the number sense assessment.
- The administrator of the number sense assessment delivered the instructions correctly and consistently to all classes.

Ethical Considerations

Approval to conduct this study with human subjects was obtained from the Institutional Review Board of Ferris State University in Big Rapids, Michigan, as shown in Appendix E. Additionally, the research was approved by the Vice President of Academic Affairs at Muskegon Community College in Muskegon, Michigan, where the research was conducted, as shown in Appendix E. Although participation in this study did not put the subjects at risk, several measures were adopted to ensure complete protection of all participants.

First, prior to participation, all students were provided with the details of the study and informed consent, and they were given the ability to opt out of having their information included.

Secondly, at the time of the study, all students being asked to participate were enrolled in mathematics courses that were in a department of which the researcher was the chair. Thus, a member of the department's office staff administered the pretest and posttest so the presence of the researcher would not influence the students’ decisions to participate. Finally, once the pretest scores, posttest scores, and final course grades were gathered and merged into one primary data set, all student identifiers were removed. Throughout the process, only the researcher had access to and reviewed the raw data.

Summary

In summary, the purpose of this study was to better understand the level of number sense of community college developmental mathematics students, how their number sense may improve throughout a typical developmental course, and if there is a relationship between their number sense and their success in that course. Because this study is seeking to identify a potential relationship between students’ number sense and their success in community college developmental mathematics courses, a correlational quantitative methodology was used. The study was conducted with students enrolled in developmental mathematics courses at a medium-sized midwestern community college in the fall of 2021. The data collected included a measure of students’ number sense at the beginning and end of their mathematics courses, the mathematics courses in which they were enrolled, and their final mathematics course grades. Several approaches were employed in the design of the study to improve its reliability and ensure the conclusions were valid. These included using an adapted version of a previously tested instrument, pilot testing the instrument, having experts in the field review the items on the test, and maintaining consistent testing conditions. Additionally, statistical tests were conducted to confirm aspects of the instrument’s reliability and validity met generally accepted standards for social science research. The following chapter shares the results of the data collection and the researcher's analyses and interpretations of those results.

CHAPTER 4: RESULTS AND ANALYSES

Introduction

Students who place and enroll in developmental mathematics courses at a community college are less likely to persist toward completing a college degree than those who arrive ready for college-level coursework (Burley et al., 2009). Although this problem has received considerable attention in recent years from a multitude of organizations, most of the approaches to improving student outcomes have focused on changing institutional structures, and few have had a significant impact on students' retention and completion rates (Rutschow et al., 2022). The purpose of this study was to explore the problem by investigating community college developmental mathematics students' number sense and how it may be associated with their academic success. This quantitative study was designed to answer the following research questions:

1. What level of number sense do community college developmental mathematics students have?
2. Is there a difference between the number sense of students enrolled in different developmental mathematics courses?
3. Does completing a traditional community college developmental mathematics course improve students' number sense?
4. Do students with greater number sense have better academic success in community college developmental mathematics courses?

The data collected included a measure of students' number sense at the beginning and end of their mathematics courses, the mathematics courses in which they were enrolled, and their final grades in their mathematics courses during the fall semester of 2021. The students' number sense was measured using a number sense assessment, which was adapted from other researchers' work (I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016; Yang, 2003), as both a pretest and a posttest. Students' final course grades were acquired from the college's student records database. The remainder of this chapter provides the results of the data collection process along with the researcher's analysis for each research question.

Research Question 1

The first question asked: What level of number sense do community college developmental mathematics students have? Research Question 1 was examined using descriptive quantitative statistics. Number sense was measured using a 37-question multiple-choice number sense assessment. Correct answers were assigned a score of 1, while incorrect answers and those that were not answered were assigned a score of 0. Thus, the minimum possible score was 0 and the maximum possible score was 37.

Results

A total of 165 students participated in the number sense pretest. Scores ranged from 3–34 and the overall mean was 19.23 (*SD* = 6.32). These data were disaggregated to examine the number sense of students based on the course in which they were enrolled, the results of which are shown in Table 2.

Table 2: Number Sense Pretest Results Disaggregated by Course

COURSE	*n*	RANGE	*M*	*SD*
Math 036a	40	3–29	15.87	5.50
Math 038	49	9–34	20.59	6.10
Math 040	53	7–34	20.00	6.55
Math 097	23	10–32	20.39	5.87

Analysis

For all students, the mean score of 19.23 translates to approximately a 52% success rate on the number sense pretest. Other researchers who have conducted similar studies have classified such results as low or poor (R. Reys et al., 1999; Singh et al., 2019; Yang & Sianturi, 2021). However, it is important to differentiate between the populations being studied. In some studies, the subjects were preservice teachers enrolled at a university (Singh et al., 2019), and thus, there was an expectation for them to have a higher level of number sense. The students examined in this study were assigned to developmental mathematics courses, an indicator that they lack some foundational mathematical skills, so it is reasonable for them to not demonstrate a high level of number sense. Overall, the students demonstrated a level of number sense that is to be expected. However, when examining the results of the pretest by course, the students in Math 036a stand out as having lower number sense than those in the other courses. Their mean score of 15.87 is approximately a 43% success rate, which, based solely on observation, appears to be significantly lower than the mean of the other three courses, which each have a mean success rate between 54% and 56%. Further analysis of these differences is explored in Research Question 2.

Given these results, we must consider how these results may have been affected by the study's delimitations. For instance, students could opt out of having their results included, so certain personal factors, such as their own mathematical confidence, may have led them to decline participation. If, for this reason, students with lower number sense were less likely to participate, this may have skewed the true number sense of the population. Additionally, students' number sense was tested using a single multiple-choice test, which can only examine their ability to identify correct answers within a given time constraint. While the questions were designed to capture students' ability to use number sense strategies, as opposed to traditional written calculations, an incorrect response did not imply they were not using number sense approaches (Yang & Lin, 2015). Follow-up interviews were not conducted and could have revealed a deeper understanding of students' thinking.

Research Question 2

The second question asked: Is there a difference between the number sense of students enrolled in different developmental mathematics courses? To examine this question, a one-way ANOVA test was conducted to examine the differences between the courses. The independent variable was the mathematics course in which the students were enrolled, a nominal-level measurement. The dependent variable, measured by the number sense pretest, was students' number sense levels at the beginning of the semester, which is an interval-level measurement.

Results

The one-way ANOVA test assumes homogeneity of variances across groups and that each group is approximately normally distributed (Maciocha, 2012). To verify these assumptions were met, Levene's test was conducted to assess the variances of number sense pretest scores across groups, and the Shapiro-Wilk test was conducted to check for normality. Results of Levene's test suggest that the variances for number sense pretest scores were equivalent, $F(3, 161) = .944$, $p = .421$. Additionally, results of the Shapiro-Wilk test, as shown in Table 3, identify a p value greater than .05 for each course, a non-statistically significant result, suggesting that the pretest scores are normally distributed for each course. Given the results of Levene's test and the Shapiro-Wilk test, the data met the assumptions for performing a one-way ANOVA test.

Table 3: Shapiro-Wilk Test of Normality for Pretest Scores Disaggregated by Course

COURSE	*W*	*df*	*p*
Math 036a	.976	40	.555
Math 038	.975	49	.386
Math 040	.979	53	.467
Math 097	.978	23	.868

A one-way ANOVA test was conducted at the $p < .05$ level and a statistically significant difference was found between courses, $F(3, 161) = 5.422$, $p = .001$, $\eta^2 = 0.092$. Post-hoc comparisons were conducted to determine which course means differed significantly. The Games-Howell test for multiple comparisons was used because of the unequal sample sizes of each group. The results of this test indicate that the mean value of number sense is statistically different between students in Math 036a ($M = 15.87$, $SD = 5.50$) and Math 038 ($M = 20.59$, $SD = 6.10$), $p = .001$, 95% CI[1.49, 7.94]. Additionally, the test suggests that the mean value of number sense is statistically different between students in Math 036a ($M = 15.87$, $SD = 5.50$) and Math 040 ($M = 20.00$, $SD = 6.55$), $p = .007$, 95% CI[0.85, 7.40]. Finally, the test indicates that the mean value of number sense is statistically different between students in Math 036a ($M = 15.87$, $SD = 5.50$) and Math 097 ($M = 20.39$, $SD = 5.87$), $p = .022$, 95% CI[0.51, 8.53]. There was no statistically significant difference found between the number sense of students in Math 038, Math 040, and Math 097.

Analysis

Research Question 2 asked the following: Is there a difference between the number sense of students enrolled in different developmental mathematics courses? The null hypothesis, which was tested at the .05 significance level, was:

> H_0: There is no significant difference between the number sense of students who are enrolled in different developmental mathematics courses.

This was tested against the following alternative hypothesis:

> H_a: There is a significant difference between the number sense of students who are enrolled in different developmental mathematics courses.

Given the results of the ANOVA test, the *p* value of less than .05 suggests that we can reject the null hypothesis and conclude that there is a statistically significant difference between students' numbers sense based on the courses in which they are enrolled. However, the partial eta squared value, $\eta^2 = 0.092$, which is generally accepted as a medium-to-large effect size (Richardson, 2011), suggests that while a statistically significant difference exists between the number sense of students enrolled in different courses, the effect is moderate. Finally, based on the results of the Games-Howell test, we can conclude that students enrolled in Math 036a have significantly lower number sense than students enrolled in the other developmental mathematics courses. Based on the researcher's experience, this finding is not surprising. The objectives of Math 036a are primarily topics found in basic arithmetic, as shown in Appendix D. Considering the assumption that students were placed into and enrolled in the correct courses, one could conclude that students enrolled in Math 036a have not mastered these skills of basic arithmetic. Thus, when acknowledging that number sense is a crucial foundational skill for mastering arithmetic (Anghileri, 2006; Baroody, 1985; Dowker, 2005; McIntosh et al., 1992) and has remained a point of emphasis at the K–12 level by the National Council of Teachers of Mathematics (NCTM; 1989, 2000), it follows that students with the lowest number sense would tend to be placed into Math 036a, the course that is intended to improve arithmetic abilities.

The finding that no significant difference exists between the number sense of students enrolled in Math 038 and Math 040 aligns with the findings of Stigler et al. (2010), who observed very few differences in the reasoning ability of students who placed into a prealgebra course compared to those who placed into an elementary algebra course. The variation in placement between the students in these two courses could have been a difference in their ability to recall mathematical procedures and not a difference in number sense.

Research Question 3

The third question asked: Does completing a traditional community college developmental mathematics course improve students' number sense? The question was examined using inferential statistics. The independent variable was the completion of a traditional developmental mathematics course. Completion of a course indicates that a student did not withdraw from the course and received a final letter grade at the conclusion of the semester as measured on a standard 0–4.0 scale. The dependent variable, students' number sense, was measured at the beginning of the semester by the pretest used in Research Question 1. The results of the pretest were compared to the students' number sense at the end of the semester, as measured by a posttest. Both the pretest and the posttest were conducted using the same instrument. Only students who completed the pretest and the posttest were included in this portion of the study. Paired samples *t* tests were used to determine if completing a developmental mathematics course improved students' number sense. The analyses were conducted for all students who completed the pretest and the posttest, and the results were disaggregated by each of the four developmental mathematics courses.

Results

All students. For all students, descriptive statistics were computed for both the number sense pretest and the posttest. There were 100 students included in this analysis. The scores of the number sense pretest ranged from 7–34 with a mean of 20.25 (SD = 5.99). The posttest scores ranged from 7–35 with a mean of 22.35 (SD = 6.72).

Both variables were examined for normality using the Shapiro-Wilk test, as a normal distribution is a requirement of a paired samples *t* test (Maciocha, 2012). Both the pretest scores, $W(100) = .985$, $p = .293$, and the posttest scores, $W(100) = .981$, $p = .161$, followed a normal distribution. Thus, a paired samples *t* test was conducted at the $p < .05$ level to compare students' number sense at the beginning and end of their developmental mathematics courses. There was a significant difference found between the pretest scores ($M = 20.25$, $SD = 5.99$) and posttest scores ($M = 22.35$, $SD = 6.72$), $t(99) = 5.25$, $p < .001$, $g = 0.52$, 95% CI[1.31, 2.89].

Disaggregated by course. Descriptive statistics were calculated for the number sense pretest and posttest scores and disaggregated by each of the four developmental mathematics courses, as shown in Table 4.

Table 4: Descriptive Statistics

COURSE	*N*	RANGE	*M*	*SD*
Math 036a				
Pretest Score	15	9–29	18.80	5.28
Posttest Score	15	13–34	21.40	5.69
Math 038				
Pretest Score	38	9–33	20.58	5.89
Posttest Score	38	7–35	22.82	7.40
Math 040				
Pretest Score	34	7–34	20.65	6.65
Posttest Score	34	8–35	21.74	6.48
Math 097				
Pretest Score	13	10–28	19.92	5.59
Posttest Score	13	12–33	23.69	6.75

To ensure the data satisfy the assumptions for a *t* test, the pretest and the posttest scores for each course were examined for normality using the Shapiro-Wilk test. The results of the

Shapiro-Wilk test indicate that the pretest scores and posttest scores for each course follow a normal distribution, as shown in Table 5.

A paired samples *t* test was conducted at the $p < .05$ level to compare students' number sense pretest and posttest scores for each course. For students in Math 036a, there was a significant difference found between pretest scores ($M = 18.80$, $SD = 5.28$) and posttest scores ($M = 21.40$, $SD = 5.69$), $t(14) = 2.39$, $p = .031$, $g = 0.60$, 95% *CI* [0.27, 4.93]. For students in Math 038, there was a significant difference found between pretest scores ($M = 20.58$, $SD = 5.89$) and posttest scores ($M = 22.82$, $SD = 7.40$), $t(37) = 3.32$, $p = .002$, $g = 0.53$, 95% *CI* [0.87, 3.60]. For students in Math 040, there was no significant difference found between pretest scores ($M = 20.65$, $SD = 6.65$) and posttest scores ($M = 21.74$, $SD = 6.48$), $t(33) = 1.54$, $p = .132$, $g = 0.26$, 95% *CI* [-0.34, 2.52]. For students in Math 097, there was a significant difference found between pretest scores ($M = 19.92$, $SD = 5.59$) and posttest scores ($M = 23.69$, $SD = 6.75$), $t(12) = 5.87$, $p < .001$, $g = 1.58$, 95% *CI* [2.37, 5.17].

Table 5: Shapiro-Wilk Test of Normality on Pretest and Posttest Scores by Course

COURSE	PRETEST SCORES			POSTTEST SCORES		
	W	*df*	*p*	*W*	*df*	*p*
Math 036a	.973	15	.896	.954	15	.583
Math 038	.971	38	.427	.948	38	.075
Math 040	.970	34	.451	.990	34	.984
Math 097	.968	13	.864	.929	13	.329

Analysis

Research Question 3 asked: Does completing a traditional community college developmental mathematics course improve students' number sense? This question was examined using inferential statistics. The null hypothesis, which was tested at the .05 significance level, was:

> H_0: As a result of completing a traditional community college developmental mathematics course, there will be no significant increase in students' number sense.

This was tested against the following alternative hypothesis:

> H_a: As a result of completing a traditional community college developmental mathematics course, there will be a significant increase in students' number sense.

Based on the results of the paired samples *t* test of all students who completed the pretest and the posttest, we can reject the null hypothesis and be confident that as a result of completing a traditional community college developmental mathematics course, there will be a significant increase in students' number sense. However, with a Hedges' *g* effect size of just 0.52, this increase can be considered moderate (J. Cohen, 1988).

We can make several conclusions at the course level, based on the results of the paired samples *t* tests. We can reject the null hypothesis and conclude that, as a result of completing Math 036a, there will be a significant increase in students' number sense. The Hedges' *g* effect size of 0.60 suggests this difference can be considered moderate (J. Cohen, 1988). We can reject the null hypothesis and conclude that, as a result of completing Math 038, there will be a significant increase in students' number sense. The Hedges' *g* effect size of 0.53 suggests this difference can be considered moderate (J. Cohen, 1988). We fail to reject the null hypothesis that, as a result of completing Math 040, there will be a significant increase in students' number sense. Finally, we can reject the null hypothesis and conclude that, as a result of completing Math 097, there will be a significant increase in students' number sense. The Hedges' *g* effect size of 1.58 suggests this difference can be considered very large (Sawilowsky, 2009).

Based on the researcher's experience, the results of the paired samples *t* tests are not surprising. Math 040 is a standard beginning algebra course, and the objectives have little association with the components of number sense, as shown in Appendix D. Thus, it is reasonable that no significant increase was observed in Math 040 students' number sense.

Of the three courses that did show a statistically significant increase in students' number sense, Math 038 had the smallest effect size, at 0.53. Math 038 emphasizes topics that introduce students to algebra, some of which are related to arithmetic and numerical concepts, as shown in Appendix D. Thus, there is a moderate connection between the course objectives and the components of number sense. Therefore, the Math 038 effect size finding is reasonable.

Math 036a had the next largest effect size, at 0.60. Given that Math 036a primarily focuses on topics of arithmetic, as shown in Appendix D, it is reasonable that students' number sense would improve throughout the course. However, based on the researcher's experience, the course is generally taught in a way that emphasizes standard algorithmic procedures more than number sense. Considering the similarities of the course objectives to the components of number sense, the moderate effect size suggests an opportunity exists to adopt teaching approaches that facilitate greater student growth in number sense.

Finally, Math 097 had the greatest effect size, at 1.58, which again is not surprising. This course was designed to improve students' mathematics literacy and has objectives that are closely associated with the components of number sense, as shown in Appendix D. Much of the course content is application based and explores foundational mathematical relationships. Unlike the other courses, it was designed to attend less to standard algorithms and more to improving students' number sense.

Research Question 4

The fourth question asked: Do students with greater number sense have better academic success in community college developmental mathematics courses? The question was explored using correlation analysis. To examine this question, the independent variable was students' number sense at the beginning of the semester, an interval-level measurement, as measured by the number sense pretest. The dependent variable was the students' final grades in their developmental mathematics courses as measured on a standard 0–4.0 scale, which is an ordinal-level measurement. The students' number sense pretest scores were compared to their final course grades using a Spearman correlation. All students who did not earn a grade for their course were removed from this analysis, including those who withdrew and those who received an incomplete. The results were also disaggregated by each of the four developmental mathematics courses.

Results

All Students. For all students included in this portion of the study, descriptive statistics were computed for both the number sense pretest and the final course grades. There were 148 students included in this analysis. The scores on the number sense pretest ranged from 7–34 with a mean of 19.47 (SD = 6.19). The grade points ranged from 0–4.0 with a mean of 2.88 (SD = 1.13).

The data used to explore this research question included one interval-level variable, number sense pretest score, and one ordinal-level variable, grade point. Thus, a non-parametric test, the Spearman rank-order correlation (one-tailed) was conducted to determine whether there was a significant relationship between the two variables. The results of the test indicated a significant positive correlation between number sense pretest scores and grade point, $r_s(146) = .300$, $p < .001$.

Disaggregated by Course. Additional analysis was conducted to explore the relationship between students' number sense and their final grades at the course level. Descriptive statistics, disaggregated by course, are shown in Table 6.

Table 6: Descriptive Statistics of Number Sense Pretest Scores and Grade Points by Mathematics Course

VARIABLE	RANGE	*M*	*SD*
Math 036a (*n* = 33)			
Number sense pretest score	8–29	16.48	5.32
Grade point	0–4.0	2.68	1.31
Math 038 (*n* = 48)			
Number sense pretest score	9–34	20.44	6.07
Grade point	0–4.0	3.39	0.97
Math 040 (*n* = 47)			
Number sense pretest score	7–34	20.19	6.72
Grade point	0–4.0	2.45	1.04
Math 097 (*n* = 20)			
Number sense pretest score	10–31	20.40	5.34
Grade point	0–4.0	3.00	0.89

Additionally, the Spearman rank-order correlation (one-tailed) was conducted to determine if there was a significant correlation between students' number sense and their final grades for each individual course, the results of which are shown in Table 7. For students in Math 036a, there was a significant positive correlation found between number sense pretest scores and grade point, $r_s(31) = .558$, $p < .001$. For students in Math 038, there was not a significant relationship found between number sense pretest scores and grade point, $r_s(46) = .155$, $p = .146$. For students in Math 040, there was a significant positive correlation found between number sense pretest scores and grade point, $r_s(45) = .337$, $p = .010$. For students in Math 097, there was a significant positive correlation found between number sense pretest scores and grade point, $r_s(18) = .470$, $p = .018$.

Table 7: Spearman Rank-Order Correlations Between Grade Point and Number Sense Pretest Scores by Mathematics Course

COURSE	r_s	p
Math 036a	.558	<.001
Math 038	.155	.146
Math 040	.337	.010
Math 097	.470	.018

Analysis

Research Question 4 asked: Do students with greater number sense have better academic success in community college developmental mathematics courses? The null hypothesis, which was tested at the .05 significance level, was:

> H_0: There is no significant correlation between students' number sense and their success in community college developmental mathematics courses.

This was tested against the following alternative hypothesis:

> H_a: A significant, positive correlation exists between students' number sense and their success in community college developmental mathematics courses.

When examining this question for all students, based on the results of the Spearman test, $r_s(146) = .300$, $p < .001$, we can reject the null hypothesis and be confident that a significant, positive correlation exists between students' number sense and their success in community college developmental mathematics courses. However, with a correlation coefficient of just .300, the effect of students' number sense on the success in their courses is weak.

At the course level, based on the results of the Spearman test, for students in Math 036a, we can reject the null hypothesis and conclude that a significant positive correlation exists between number sense and success in the course. With a Spearman coefficient of .588, that relationship is moderate. For students in Math 038, we fail to reject the null hypothesis that a relationship exists between number sense and success in the course. For students in Math 040, we can reject the null hypothesis and conclude that a significant positive correlation exists between number sense and success in the course. However, with a Spearman coefficient of .337, that relationship is weak. Finally, for students in Math 097, we can reject the null hypothesis and conclude that a significant positive correlation exists between number sense and success in the course. With a Spearman coefficient of .470, that relationship is moderate.

From the researcher's experience, many factors can influence students' success in their developmental mathematics courses. Based on the findings, it appears that students' number sense may be one of those factors for Math 036a, Math 040, and Math 097. Although we found some correlations between students' number sense and success in their courses, none of those correlations were strong. However, we found two courses with a moderate correlation: Math 036a and Math 097. These results are unsurprising, as Math 036a and Math 097 have objectives closely associated with the components of number sense, unlike Math 038 and Math 040, which are more algebra-based courses. It is reasonable that students entering a semester with greater number sense would be more likely to succeed in a course that has content aligned with number sense.

Finally, we must consider several of the study's limitations and delimitations when analyzing the results of Research Question 4. Considering that this study was limited to one institution, the correlation between number sense and course success may vary at other institutions given the difference in course objectives at different colleges. Additionally, during the semester in which the research took place, the college was in the process of transitioning away from standardized placement tests and to a model in which students can self-select their mathematics placement. Thus, there was no mandated standard in place for assigning students to different levels of mathematics. Although there were guidelines and recommendations in place, there may have been students who ignored those recommendations and chose a course for which they were significantly over-prepared or under-prepared. Finally, there are many factors other than number sense that determine a student's success in a mathematics course. Non-academic factors such as childcare, work schedules, motivation, adherence to academic integrity, or health were not considered, although they may have contributed significantly to some students' lack of success.

Summary

The purpose of this study was to better understand the level of number sense of community college developmental mathematics students, how their number sense may improve throughout a typical developmental course, and if there is a relationship between their number sense and their success in that course. The findings of this study accomplished these objectives.

We now know that overall, students in the developmental mathematics courses had approximately a 52% success rate on the number sense pretest, which is generally considered low but is a reasonable expectation for this population. Furthermore, students in Math 036a have significantly lower number sense than students in Math 038, Math 040, and Math 097. This was also expected, given the content of each course and current recommendations for placement. We also now know that Math 040 was found to have no effect on improving students' number sense, Math 036a and Math 038 were found to have a moderate effect, and Math 097 was found to have a very large effect. Again, these results were not surprising, given the objectives of those courses. Finally, we now know that overall, number sense has a significant but weak positive correlation with students' success in developmental mathematics courses. At the course level, no correlation was found between number sense and success in Math 038, a weak positive correlation was found in Math 040, and a moderate positive correlation was found in Math 036a and Math 097. These results were unsurprising, knowing that many factors can influence students' success in developmental mathematics courses. A discussion of the larger implications of these findings and recommendations for future research is included in the following chapter.

CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

Introduction

This chapter provides a summary of the study; a discussion of the findings, implications, and recommendations; and recommendations for future research.

Summary of Study

Chapter 1 introduced the context of the problem and purpose of the study. Through an introduction to the literature, we learned that students who place into and enroll in developmental mathematics courses at a community college are less likely to persist toward completing a college degree than those who arrive prepared for college-level coursework (Burley et al., 2009). Although this problem has received considerable attention in recent years, few interventions have had a significant impact on students' retention and completion rates (Rutschow et al., 2022). Chapter 1 also introduced the construct of number sense, a crucial foundational skill for all learners of mathematics (Maclellan, 2012; NCTM, 1989, 2000), although one with little investigation at the community college level. Given this context, the purpose of this study was to investigate community college developmental mathematics students' number sense and how it may be associated with their academic success. The study was designed to answer the following research questions:

1. What level of number sense do community college developmental mathematics students have?
2. Is there a difference between the number sense of students enrolled in different developmental mathematics courses?

3. Does completing a traditional community college developmental mathematics course improve students' number sense?
4. Do students with greater number sense have better academic success in community college developmental mathematics courses?

Chapter 2 provided a comprehensive examination of the existing literature on developmental mathematics in community colleges, recent efforts to improve developmental mathematics outcomes, foundations of the number sense construct, number sense studies of children and adolescents, and number sense studies of college students. Notably, Chapter 2 introduced I. Whitacre et al.'s (2020) description of mature number sense—that it is learned, involves habits of mind, and is studied in populations ranging from elementary students to adults—which was used as a basis for the theoretical framework of this study.

Chapter 3 explained the quantitative research methods that were used to address the four research questions. The participants were students who were enrolled in one of the following four courses at Muskegon Community College in the fall of 2021: Math 036a (Basic Math), Math 038 (Prealgebra), Math 040 (Beginning Algebra), and Math 097 (Math Literacy). The data collected included a measure of students' number sense at the beginning and end of their courses, the courses in which they were enrolled, and their final course grades. The students' number sense was measured using an assessment tool adapted from other researchers' previous work (I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016; Yang, 2003) as both a pretest and a posttest. Students' final course grades were acquired from the college's student records database.

Chapter 4 presented the results and analyses of the statistical tests that were used to answer each research question. The data were analyzed using descriptive statistics, a one-way ANOVA test, paired samples *t* tests, and a Spearman correlation. The results of these analyses suggested the following:

- Community college developmental mathematics students have low number sense.

- There is a significant difference between the number sense of students enrolled in different developmental mathematics courses.
- Students' number sense improves while completing some developmental mathematics courses but not others.
- Greater number sense is correlated with higher grades in some developmental mathematics courses.

In the next section is a discussion of the relevance and implications of these findings and how they may be meaningful to practitioners.

Discussion of Findings, Implications, and Recommendations

We have observed, from exploring the research questions in Chapter 4, that developmental mathematics students at Muskegon Community College demonstrate a level of number sense that other researchers have classified as low or poor (R. Reys et al., 1999; Singh et al., 2019; Yang & Sianturi, 2021). Specifically, students enrolled in Math 036a (Basic Math) have weaker number sense than those in Math 038 (Prealgebra), Math 040 (Beginning Algebra), and Math 097 (Math Literacy). Furthermore, this study suggests that for many students enrolled in Math 036a, Math 038, and Math 097, their number sense improves throughout the semester. Finally, we observed that students with greater number sense are more likely to have higher course grades in Math 036a, Math 040, and Math 097. The results of this study have several implications, particularly for faculty who teach these courses. This section explains what the findings of each research question mean in the context of this study's purpose, what the implications of the findings mean for practitioners, and how these findings compare to the findings from other studies.

Research Question 1

Research Question 1 explored the overall number sense of all developmental mathematics students at Muskegon Community College, and the results suggest that these students have a low level of number sense. Many other researchers found similarly low levels of number sense in other populations using a variety of instruments for data collection and research approaches (Akkaya, 2016; Ali, 2014; Facun & Nool, 2012; Gürefe et al., 2017; Menon, 2004; R. Reys et al., 1999; Singh et al., 2019; Tsao, 2005; Yang, 2007; Yang et al., 2009; Yang & Huang, 2004; Yang & Sianturi, 2019, 2021). These studies included students in primary grades, middle grades, high school, and universities both within the United States and internationally. Two additional studies, Givvin et al. (2011) and Steinke (2017), explored community college developmental mathematics students' conceptual understanding of mathematics and number sense, respectively; but neither attended to the construct of number sense as it is commonly described in the literature. However, the low number sense of developmental mathematics students found in the current study does align with and confirm the findings of both Givvin et al. and Steinke.

In contrast to students in K–12 settings or preservice teachers at universities, a unique characteristic of students in community college courses is that they have traditionally been assigned to their courses using placement measures such as standardized tests. There is, however, a growing movement to empower students by allowing them to select their own starting level based on the college's recommendation, often referred to as guided or directed self-placement (Kosiewicz & Ngo, 2020). According to Stigler et al. (2010), standardized tests typically assess students' abilities to solve mathematics problems using common algorithmic procedures. Number sense, conversely, is considered to describe one's ability to solve problems using nonroutine strategies, that is, without using common algorithmic procedures. The students in this study placed into the lowest mathematics courses, an indication of their lack of procedural skills. Additionally, their low level of number sense, as observed in this study, suggests they also may lack the ability to solve problems in nonroutine ways. The implications of this finding confirm what many developmental mathematics faculty may already assume: Most students enter their courses with an extremely weak mathematical foundation. However, to support students' successful transition to college-level mathematics courses, it is beneficial for faculty to have a more complete picture of where their students' weaknesses are. Knowing that the students not only lack procedural skills but also have poor number sense may encourage faculty to examine their own approaches to teaching their courses. For the faculty who find value in this knowledge, number-sense-based instruction has been shown to be effective at developing the number sense of students at all levels (Markovits & Sowder, 1994; Ulusoy, 2020; I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016; Yaman, 2015; Yang, 2003; Yang et al., 2004; Yang & Tsai, 2010; Yang & Wu, 2010). Specifically, as noted by Yang (2003), number sense can be developed in the classroom through an instructional design that "encourages communication, exploration, discussion, thinking, and reasoning" (p. 132), rather than one that strictly emphasizes procedures.

The importance of developing students' number sense, particularly for those whose number sense is weak, has been demonstrated throughout the literature (Gersten & Chard, 1999; Maclellan, 2012; National Council of Teachers of Mathematics, 1989, 2000). Notably, Gray and Tall (1994) observed that students skilled in certain components of number sense are able to efficiently manipulate prior knowledge to derive new knowledge. If developmental mathematics faculty primarily focus their teaching on algorithmic procedures and do not attend to developing number sense, then, as noted by Gray and Tall, their students may continue to view mathematics as the unending stacking of complex processes. This mentality further increases the difficulty of learning mathematics as the students encounter additional challenges, which the developmental mathematics students inevitably will experience as they progress to college-level mathematics coursework.

Research Question 2

The findings of Research Question 2 suggest that the students in Math 036a (Basic Math), the lowest mathematics course into which students can place, have significantly lower number sense than the students in Math 038 (Prealgebra), Math 040 (Beginning Algebra), and Math 097 (Math Literacy). In practicality, this may have several moderately important implications. For faculty, knowing that students in Math 036a enter the course with a greater deficiency in number sense compared to other students emphasizes the importance of intentionally integrating number-sense-based teaching strategies. This is no different from the recommendation stated in the previous section on Research Question 1.

What is intriguing about the results of Research Question 2 is that no statistical difference in number sense was found between the students enrolled in Math 038, Math 040, and Math 097. This finding aligns with that of Stigler et al. (2010) who observed similar levels of number sense among students enrolled in a prealgebra course, like Math 038, and those enrolled in a beginning algebra course, like Math 040. For college administrators and faculty leaders, this exposes an opportunity to be explored regarding mathematics placement policies and course objectives. As described in Chapter 1, Muskegon Community College redesigned the developmental mathematics prerequisites in 2021 to help students enroll in mathematics courses that better align with their degree pathways. Math 097 is a single course prerequisite in the quantitative reasoning pathway. In the STEM pathway, Math 036a is a prerequisite to Math 038, which is a prerequisite to Math 040. However, because the students in Math 038 and Math 040 have statistically the same level of number sense, the difference in their mathematical abilities may be observed by what they can demonstrate on standardized tests, which, according to Stigler et al. (2010), generally emphasize procedural knowledge over number sense. Considering this context alongside the needs and policies of individual institutions, practitioners should explore strategies to better assess their students' mathematical abilities for placement, realign their course objectives, or condense their developmental mathematics course sequence. For example, at Muskegon Community College, considering that Math 038 and Math 040 are consecutive courses in the STEM pathway and their students have the same statistical level of number sense, it may be beneficial to combine the two courses into one. Most importantly, colleges must develop approaches that help ensure students are enrolling in courses that best meet their needs and address the specific gaps in their foundational mathematical abilities.

Research Question 3

Research Question 3 indicates that students' number sense improved throughout their time in Math 036a (Basic Math), Math 038 (Prealgebra), and Math 097 (Math Literacy). Math 097, the course in which the largest statistical effect size was observed, seemed to have the greatest impact on improving students' number sense. Additionally, no increase in number sense was observed for the students in Math 040 (Beginning Algebra). At a course level, these findings are significant if a goal of the course is to improve students' number sense. As shown in Appendix D, Math 097 is the only course of the four with objectives that are directly associated with the components of number sense. Thus, it was expected that Math 097 facilitated the most significant increase in number sense among its students. However, these results must be examined within the context of the problem this study is addressing, which is that developmental mathematics students struggle to complete college degrees. Improved number sense at the course level is only significant in relation to this problem if it leads to improved student success in subsequent higher-level mathematics courses. Thus, it may be valuable to consider the importance of number sense in subsequent courses. For instance, following the successful completion of Math 036a, students then enroll in Math 038. If we knew that students with a greater level of number sense were more successful in Math 038, then it would make sense to put a greater emphasis on improving students' number sense in Math 036a. Thus, the results of Research Question 3 may have more value when paired with the results of Research Question 4, which explored the relationship between students' number sense and their course outcomes.

There have been consistent results within the literature regarding students' number sense improving when number-sense-based instruction was implemented (Markovits & Sowder, 1994; Ulusoy, 2020; I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016; Yaman, 2015; Yang, 2003; Yang et al., 2004; Yang & Tsai, 2010; Yang & Wu, 2010). However, these studies often examined different populations than the current study and there was often a specific intervention that was implemented to attempt to improve students' number sense. In this study, no such intervention was conducted. This research question was unique compared to those in the broader literature as it was designed simply to determine if students' number sense improved given the traditional course objectives and teaching approaches. Perhaps a reason for the improvement of number sense in several of the courses examined in Research Question 3 was that, compared to students in other studies, many community college students are considered non-traditional. That is, they may not have come to college directly from high school and thus, there has been time for their mathematical skills to atrophy. Perhaps simply being exposed to formal mathematics after time away re-engaged some of their pre-existing but dormant number sense, even if the instruction was not number-sense-based. At this point, however, there have been no studies that examine how one's number sense may change over time outside of formal schooling.

Regarding the students in Math 040 experiencing no increase in number sense, this was consistent with themes found in the literature. Math 040 is a beginning algebra course that, in the researcher's experience, relies the most on procedures of the four courses included in this study. A common theme found in the literature was that procedural-based approaches to teaching mathematics may hinder the development of number sense (Aperapar & Hoon, 2011; Givvin et al., 2011; Tsao, 2005). Thus, the findings of Research Question 3 align with those of previous studies.

Research Question 4

Research Question 4 explored the relationship between students' number sense and their course outcomes. Specifically, the question compared the students' number sense at the beginning of the course with their final grades at the end of the course. A significant, positive correlation was found between these variables for the students enrolled in Math 036a (Basic Math), Math 040 (Beginning Algebra), and Math 097 (Math Literacy), but not Math 038 (Prealgebra). These results support previous studies that found students with greater number sense generally have better mathematics achievement (Bütüner, 2018; Mohamed & Johnny, 2010; Singh et al., 2019; Steinke, 2017; Yang et al., 2008).

Given these findings, we may infer that students who enter those courses with a higher level of number sense are more likely to complete the course with a higher grade. Thus, there may be benefit in establishing programs that can help students improve their number sense prior to entering certain mathematics courses. We already observed in Research Question 3 that students' number sense improves throughout their time in Math 038, a course that is a prerequisite for Math 040. Therefore, faculty charged with coordinating and teaching Math 038 should work to implement pedagogical approaches that have shown promise in improving students' number sense.

Another implication of this study centers on student preparation. Some colleges offer bridge programs or mathematics boot camps in the summer for new incoming students (Hodara, 2013), the purpose of which are to prepare students for the college experience and help them develop the skills necessary to be successful once they begin their mathematics courses. Such a setting may be an opportune time to implement mathematics instruction that specifically addresses students' number sense.

Interestingly, there was a positive correlation between number sense and final course grades in Math 040, although, as shown in Research Question 3, Math 040 was not associated with any statistically significant increase in students' number sense. This may suggest that integrating strategies to improve students' number sense throughout Math 040 could potentially lead to better course outcomes. This may be an opportunity for further investigation.

Recommendations for Future Research

The broad problem that this study set out to investigate is that community college developmental mathematics students struggle to earn college credentials. This study explored the problem by examining the number sense of these students. The findings of this study have contributed to the knowledge base of both the number sense literature and the literature surrounding developmental mathematics in community colleges. However, the intersection of these two areas in the literature remains incomplete, and there are several key opportunities for future research that could help further improve the knowledge base.

Addressing Limitations and Delimitations

There were multiple limitations and delimitations to this study. Addressing some of these limitations and delimitations in future research could provide additional insights into the number sense of community college developmental mathematics students. First, this study was limited to students in developmental mathematics courses. In the landscape of higher education, there have been several studies that examined the number sense of preservice teachers enrolled in college-level mathematics courses at universities. Yet there have been very few studies at community colleges, even though universities and community colleges generally serve somewhat different student populations (A. M. Cohen et al., 2013). Thus, additional research exploring the number sense of students who place directly into college-level mathematics courses at community colleges would be beneficial, as improved number sense may be a skill that benefits students at all levels of the mathematics continuum.

Secondly, the purpose of developmental mathematics courses is to prepare students for those college level-courses. That is, students in developmental mathematics courses should advance to enrolling in a college-level mathematics course in which they earn credit toward their degree. Thus, conducting a longitudinal study to track students' cumulative change in number sense over multiple semesters as they progress from developmental mathematics to college-level courses would be valuable. Such methodology may include assessing their number sense at multiple points to determine how well different courses improve their number sense, how well they retain their number sense, and how their number sense may be connected to broader student success outcomes such as success in subsequent mathematics courses, persistence from one semester to the next, and degree attainment.

Third, this study used a single multiple-choice quantitative test to assess students' number sense. Other researchers investigating number sense have used additional approaches such as tiered surveys (Yang, 2019; Yang & Lin, 2015; Yang & Sianturi, 2019, 2021; Yang & Tsai, 2010) and follow-up interviews (Almeida et al., 2016; Bütüner, 2018, 2018; Givvin et al., 2011; Markovits & Sowder, 1994; Menon, 2004; Mohamed & Johnny, 2011; R. E. Reys & Yang, 1998; Şengül & Gülbağcı, 2012; Tsao, 2005; I. M. Whitacre, 2012; Yaman, 2015; Yang, 2003, 2007; Yang et al., 2004, 2004; Yang & Wu, 2010) to better understand students' reasoning. Integrating tiered surveys and qualitative methodologies such as interviews would help reveal the students' reasonings, expose specific areas of foundational weaknesses, and add a richness to the data.

Finally, based on the researcher's experience, there are many factors that can lead to a student's success or lack thereof in a college mathematics course. These may include such factors as age, years since last mathematics course, family experience with college, hours spent working, family obligations, motivation, race and ethnicity, growth mindset versus fixed mindset, and success in high school. None of these factors were considered in this study's methodology. However, a study that controls for these or other factors related to a student's success in college may help to better quantify the effect of number sense on mathematics course outcomes.

Other Recommendations

Besides the recommendations inspired by this study's limitations and delimitations, there are other valuable opportunities for future research in this area. The findings of this study suggest that final course grades are correlated with number sense at the start of the semester for students in Math 036a, Math 040, and Math 097. What we do not know is if improvement in number sense throughout the semester is correlated with students' course outcomes. Thus, research that explores the relationship between students' change in number sense and their course outcomes would be valuable. If it can be shown that an increase in number sense leads to improved course outcomes, then an argument could be made for investing in training for faculty in number-sense-based instruction.

Additionally, many researchers studied preservice teachers and found varying levels of number sense (Aktaş & Özdemir, 2017; Almeida et al., 2016; Tsao, 2004, 2005; Yang, 2007; Yang et al., 2009). Therefore, it stands to reason that teachers of developmental mathematics may also have varying levels of number sense, although there has been no investigation of this in the literature. If teachers have varying levels of number sense, then some may be more inclined to use procedural-based approaches to teaching mathematics, which can hinder the development of some students' number sense (Aperapar & Hoon, 2011; Givvin et al., 2011; Tsao, 2005). Therefore, another area of potentially valuable research would be to explore the number sense and teaching approaches of community college developmental mathematics faculty. Given the results of those findings, colleges may be incentivized to provide professional learning activities that help to educate mathematics faculty about number sense and the benefits of teaching methods that promote number sense.

Another intriguing area for future research is the relationship between number sense and students' self-efficacy and growth mindset with respect to mathematics. In the researcher's experience, many students lack the belief that they can be successful in college mathematics courses, and any opportunity to improve their self-efficacy with respect to mathematics could be beneficial to their success, with improved number sense being no exception.

Finally, number-sense-based instruction has been shown to be effective at developing the number sense of students at many levels (Markovits & Sowder, 1994; Ulusoy, 2020; I. M. Whitacre, 2012; I. Whitacre & Nickerson, 2016; Yaman, 2015; Yang, 2003; Yang et al., 2004;

Yang & Tsai, 2010; Yang & Wu, 2010). However, these studies were only conducted in the K–12 setting or with preservice teachers at universities. Now that we better understand the number sense of developmental mathematics students, further research should explore the effect that number-sense-based teaching strategies have on improving the number sense of this population. Additionally, as referenced in Chapter 2, many colleges are now adopting a corequisite approach for their developmental mathematics courses (Logue et al., 2019; Park et al., 2018; Ran & Lin, 2019). Integrating instructional approaches intentionally designed to improve students' number sense in those corequisite courses may be an effective strategy to improve the outcomes in their gateway college-level mathematics courses, and the impact of such an intervention may be worth future research.

Personal Reflection

On a personal note, I have observed the landscape of developmental mathematics change significantly over the past decade. While I find value in most of the changes that have transpired, many community colleges still offer multiple developmental mathematics courses, the most basic of which teach concepts of arithmetic, including calculations with decimals, fractions, and percentages. While these concepts are integral for success in later mathematics courses, I suspect many students who demonstrate proficiency with them in the developmental courses do so somewhat superficially. This may be observed when students excel in the developmental courses yet struggle when they reach College Algebra. New knowledge must be built upon a stable foundation. The theme of the outcomes of this study is that improving students' numbers sense helps solidify their mathematical foundation, allowing them to learn more effectively and achieve greater success in all of their mathematics courses.

I have found great value in both the findings of this study as well as the process of conducting the research. In my 15 years of full-time teaching experience at a community college, I have observed that students in developmental mathematics courses have often had demoralizing mathematical experiences in their pasts. Many come with the baggage of having been told by someone—perhaps a parent, teacher, or counselor—that they are not college material, that they will never succeed in mathematics, or that they are just not a math person. Many developed a fear of mathematics during their K–12 experiences, often in elementary school when timed quizzes over operations were emphasized or during middle school when variables were first introduced. Many have shared that, at some point, mathematics changed from something that was fun into something to be feared. Thus, they generally come to my classes believing they are incapable of mastering any form of mathematics. Through my own professional learning, as someone who primarily teaches students in developmental mathematics courses, I have discovered that I must first work at dismantling those beliefs and helping students develop the mindset that they are capable of learning mathematics. Furthermore, I have observed that, even when students have begun to develop mathematical confidence, if they encounter challenging topics taught using traditional teaching methods—often standard algorithms that they have struggled with in the past—they often revert to doubting their capacity to succeed. This is where I remain intrigued and excited about the use of number-sense-based approaches to teaching, which do not emphasize standard algorithms. Although this study did not introduce number-sense-based teaching practices into the developmental mathematics courses, I see this as an excellent opportunity to improve the way in which we can improve our students' mathematics experiences and ultimately, their success in college.

Summary

The problem this study attempted to illuminate is that students who enter community colleges unprepared for college-level mathematics are much less likely to complete a college degree than their mathematically-prepared counterparts (Burley et al., 2009). In recent years, policymakers and college leaders have directed changes to developmental education that include structural interventions such as acceleration, corequisites, emporium models, placement using high school grade point averages, and elimination of developmental coursework altogether. The lack of mathematical preparation for many students entering community college is a problem, but so is the institution's ability to guide those students to completion. In the researcher's experience, community college leaders often speak about the importance of equity and meeting students where they are academically, yet few of the policy changes and interventions are truly designed to meet struggling mathematics students where they are. For many students, the researcher has observed that the change to structure may simply shift, mask, or ignore the problem that they lack a strong foundation of mathematical understanding.

This study approached the problem not by addressing the systemic structure of developmental mathematics but by focusing on one aspect of what it means to effectively build mathematical knowledge upon a solid foundation. Specifically, this study explored the problem by investigating the number sense of community college developmental mathematics students, how their number sense may improve throughout a typical developmental course, and if there is a relationship between their number sense and their success in that course. The findings of this study accomplished these objectives.

From exploring the research questions in Chapter 4, we observed a low level of number sense among the developmental mathematics students at Muskegon Community College, a difference between the number sense of students enrolled in different developmental mathematics courses, improvement in students' number sense while completing some developmental mathematics courses, and a positive correlation between number sense and final grades in some developmental mathematics courses.

Given the findings of this study, there is reason for intrigue. It appears that there is a relationship between number sense and student outcomes among community college developmental mathematics students. More research is necessary with community college developmental mathematics students to determine how important this relationship may be and what the long-term outcomes are of improved number sense. However, as community colleges continue to try to improve student outcomes through redesigns of developmental mathematics, faculty and college leadership must remain mindful of the foundational skills—such as number sense—that have a positive impact on students' success in college-level mathematics.

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APPENDIX A: NUMBER SENSE ASSESSMENT

Welcome! Your participation in this activity serves two purposes:

1. It is part of a standard pretest and posttest process that allows us to continuously improve our courses.
2. It is part of a doctoral research study.

Expect the entire process to take about 30–40 minutes.

The next three pages contain information and questions about your participation in the research study and instructions for completing the test. Please read the information on these pages and answer the questions that follow.

In the next section, you will complete 37 math questions **without using a pen, paper, or calculator**.

The survey is intended to investigate aspects of your mathematical thinking. The questions are not meant to be solved by written work, so please do not do any writing. Simply read each question, consider the answer options, and choose the best answer based on your knowledge, reasoning, and/or mental math. Don't worry if you're unsure about some of the responses. You can make an educated guess or just indicated that you're unsure. Please just select the response that best reflects your mental reasoning.

Each question has a time limit of 45 seconds. If you do not answer within the allotted time, it will automatically advance to the next question.

This survey is confidential. Your instructor will not be informed of your individual responses, and your responses will not affect your course grade. Please just do your best.

Thanks!

1) How many digits are in the answer to a 2-digit number multiplied by a 2-digit number?

- A) must be three digits
- B) must be four digits
- C) can be three digits or four digits
- D) can be three digits, four digits, of five digits

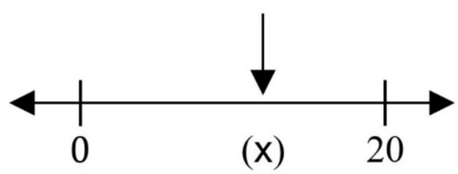
2) Compare $\frac{7}{11}$ and $\frac{7}{10}$. Which is greater?

- A) $\frac{7}{11}$
- B) $\frac{7}{10}$
- C) they are equal
- D) I'm not sure

3) Which answer is the result of 18×19 closest to?

- A) 250
- B) 350
- C) 450
- D) 550

4) Which number below is the best estimate of (x)?



- A) 8
- B) 10
- C) 12
- D) 17

5) The answer to a 3-digit number added to a 3-digit number:

- A) must be three digits
- B) must be four digits
- C) can be three digits or four digits
- D) can be three digits, four digits, or five digits

6) Compare 7.2 and 7.1987. Which is greater?

- A) 7.2
- B) 7.1987
- C) they are equal
- D) I'm not sure

7) A cat eats 30 ounces of fish every four days. How many ounces of fish does this cat eat in six days?

- A) 20
- B) 40
- C) 45
- D) 60

8) How many fractions are between $\frac{4}{7}$ and $\frac{5}{7}$?

- A) none
- B) one
- C) ten
- D) infinitely many

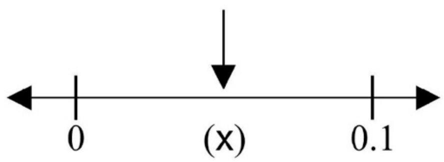
9) Which product below is the greatest?

- A) 18×17
- B) 16×18
- C) 17×19
- D) 19×15

10) Compare 3.111 and 3.1099. Which is greater?

- A) 3.111
- B) 3.1099
- C) they are equal
- D) I'm not sure

11) Which value below is the best estimate of (x)?



- A) 0.01
- B) 0.5
- C) 0.05
- D) 0.005

12) Which is the most typical weight of a male human who is 67 inches tall?

- A) 22 pounds
- B) 55 pounds
- C) 154 pounds
- D) 431 pounds

13) Which description of 145×4 is correct?

- A) greater than 450
- B) smaller than 450
- C) equal to 450
- D) I can't tell without calculating

14) This statement is true: $93 \times 134 = 12,462$

How much greater than 12,462 is the result of 93×135?

- A) 93
- B) 134
- C) 135
- D) I can't tell without calculating

15) Which description below is correct for $\frac{2}{5}$?

- A) greater than $\frac{1}{2}$
- B) equal to 2.5
- C) equal to 0.4
- D) smaller than $\frac{1}{4}$

16) Which fraction below is between $\frac{4}{5}$ and 1?

- A) $\frac{2}{3}$
- B) $\frac{3}{4}$
- C) $\frac{5}{6}$
- D) $\frac{5}{5}$

17) Which answer below is greater than 1?

- A) $\frac{2}{5} + \frac{3}{7}$
- B) $\frac{1}{2} + \frac{4}{9}$
- C) $\frac{3}{8} + \frac{2}{11}$
- D) $\frac{4}{7} + \frac{1}{2}$

18) Which description of $6\frac{2}{5} \div \frac{15}{16}$ is correct?

- A) greater than $6\frac{2}{5}$
- B) smaller than $6\frac{2}{5}$
- C) equal to $6\frac{2}{5}$
- D) I can't tell without calculating

19) Mary took a trip. She spent 5 hours traveling to her destination with an average speed of 45 miles/hour. Mary's return trip took only 4 hours. What was her *approximate* average speed on the return trip?

- A) 34 miles/hour
- B) 45 miles/hour
- C) 56 miles/hour
- D) I can't tell without calculating

20) Which answer below is the same as 0.5×840?

- A) $840 \div 2$
- B) 5×840
- C) $840 \div 5$
- D) 0.50×84

21) If $103 \times 236 = 24,308$, then $103 \times 235 = ?$

- A) 24,307
- B) 24,205
- C) 24,335
- D) 24,544

22) Which description of 87×0.09 is correct?

- A) much smaller than 87
- B) a little bit smaler than 87
- C) much greater than 87
- D) a little bit greater than 87

23) Which number can we correctly put into ( ) in the following equation?

$\frac{1}{2} \times (\ ) = \frac{4}{8}$

- A) $\frac{3}{4}$
- B) $\frac{3}{6}$
- C) 1
- D) 4

24) Which value below is equal to $1\frac{1}{4}$?

- A) 1.14
- B) 1.41
- C) 1.25
- D) 1.0

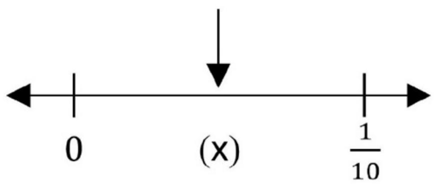
25) Which description below is correct for the result of $\frac{3}{7} + 1.5$?

- A) greater than 2
- B) smaller than 2
- C) equal to 2
- D) fractions and decimals cannot be added together

26) Which description of 245×0.98 is correct?

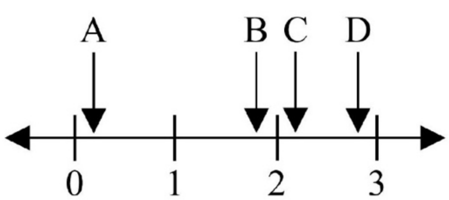
- A) greater than 245
- B) smaller than 245
- C) equal to 245
- D) I can't tell without calculating

27) Which fraction below is the best estimate of (x)?



- A) $\frac{5}{10}$
- B) $\frac{5}{100}$
- C) $\frac{1}{100}$
- D) $\frac{5}{1000}$

28) Which location in the figure can represent 2.19 best?



- A
- B
- C
- D

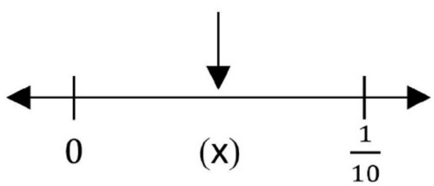
29) Which description below is correct for 9×99.99?

- A) greater than 900
- B) smaller than 900
- C) equal to 900
- D) I can't tell without calculating

30) Which description of 0.997×0.9 is correct?

- A) greater than 0.9
- B) smaller than 0.9
- C) equal to 0.9
- D) I can't tell without calculating

31) Which fraction below is the best estimate of (x)?



- A) $\frac{2}{10}$
- B) $\frac{5}{10}$
- C) $\frac{1}{20}$
- D) $\frac{2}{100}$

32) A pizza is cut into 8 equal pieces. Then each piece is cut into 3 equal pieces. How many equal pieces does the pizza have now?

- A) 3
- B) 8
- C) 11
- D) 24

33) Compare 521×5 and $520 + 521 + 522 + 523 + 524$.

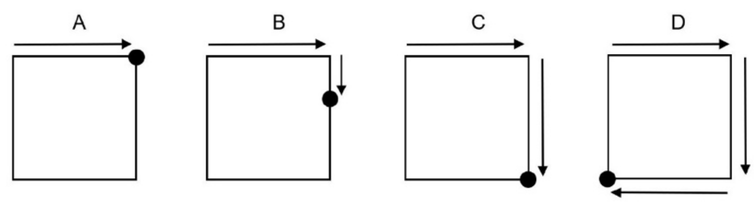
Which is greater?

- A) the result of 521×5 is greater
- B) the result of $520 + 521 + 522 + 523 + 524$ is greater
- C) they are equal
- D) I can't tell without calculating

34) Which description of $487 \div 0.99$ is correct?

- A) greater than 487
- B) smaller than 487
- C) equal to 487
- D) I can't tell without calculating

35) Sam starts off at the top left corner of a square and moves (clockwise) along it. Which figure shows the point at which Sam has walked $\frac{1}{3}$ of the way around the entire outside edge?



- A
- B
- C
- D

36) How many decimal numbers are between 9.43 and 9.44?

- A) none
- B) one
- C) ten
- D) infinitely many

37) Which answer is the result of $\frac{14}{15} + \frac{7}{8}$ closest to?

- A) $\frac{2}{3}$
- B) 1
- C) 2
- D) I can't tell without calculating

APPENDIX B: PRE-TEST SCRIPT

First of all, thank you for letting me join your class today. My name is (your name) and I am the (your title) for the College Success Center. I am here today for two reasons. The first is to administer a number sense pretest as part of our departments' regular assessment activities. We will also have you complete a posttest toward the end of the semester.

The test is intended to investigate aspects of your mathematical thinking and requires you to solve the questions with no written work or calculator. Additionally, each question is multiple choice and has a time limit of 45 seconds.

I am also here to invite you to participate in a research study that is being conducted by one of our math instructors. What this means is that you will be asked for permission by the researcher to use and study your test results and your final grade in this math course. The researcher hopes to better understand the number sense of MCC's math students with the goal of improving their success.

If you agree to have your results included, your personal information will remain completely confidential. The results will be stored in a password-protected website and document that only the researcher has access to. None of your personally identifiable information will be shared. Whether you chose to participate in the study or not will have no impact on your relationship or academic standing with the college.

In a moment, I will share a link with you that will take you to the testing website. I will then talk you through the instructions. You will be read information about what your participation in the study will involve, and you will be asked to provide your consent to include your results in the study. After that, you will complete the test. Please note, that if you are under 18, you should still complete the test, but do not agree to have your results included in the research study.

Are there any questions before we begin? (*Pause for questions*.)

Now, please open a web browser and go to the following website. (*Provide url and pause for students to complete this step*.) You should now all be at the welcome screen. As noted, your participation in this activity is part of a standard pretest and posttest process that allows us to continuously improve our courses. It is also part of a doctoral research study. Expect the entire process to take about 30–40 minutes.

The next three pages contain information and questions about your participation in the research study and the instructions for completing the test. Click Start Quiz to go to the next page. (*Pause for students to complete this step*.) Please take a moment to provide your name, student ID—if you know it—and math class. (*Pause for students to complete this step*.) When you are done, click Next Page to move on. (Pause for students to complete this step.)

Here you can read the specific information about what it means to participate in the research study. At the bottom of the page, you will choose whether or not to have your information included. Type your name and today's date (*state the date*) if you choose to participate. Once you have completed this step, you will advance to the next page, which includes instructions for completing the test.

As you read the consent information, let me know if you have any questions. If you have questions or concerns about what it means to participate in the study that I cannot answer, or if you wish to change your mind about having your results included, you may contact J.B. Meeuwenberg, whose information is provided, through the end of the semester.

Are there any questions before we start? (*Pause for questions*.)

You are now free to begin.

APPENDIX C: POST-TEST SCRIPT

Thank you for letting me join your class again. My name is (your name) from the College Success Center. Earlier in the semester I came to give a pretest and today you will be taking a posttest. The test requires you to solve the questions with no written work or calculator. Just like before, each question is multiple choice and has a time limit of 45 seconds.

You will also be asked permission again to have your results and your final grade used as part of a doctoral research study. If you agree to have your results included, your personal information will remain secure and completely confidential, and your choice to participate in the study will have no impact on your relationship with the college. On the consent page, please be sure to include your name and today's date (*state the date*) if you agree to have your results included.

As you read the consent information, let me know if you have any questions. If you have questions or concerns that I cannot answer, or if you wish to change your mind about having your results included, you may contact J.B. Meeuwenberg, whose information is provided, through the end of the semester. Again, if you are under 18, you should still complete the test, but do not agree to have your results included in the research study.

Now, please open a web browser. (*Pause and wait for all to have a browser open*.) Once I provide you with the link, please read and complete the first several pages completely before beginning the posttest. Remember, once you begin the test each question has a time limit of 45 seconds.

Are there any questions before we begin? (*Pause for questions*.)

Please visit following website and begin. (*Provide url.*)

APPENDIX D: DEVELOPMENTAL MATHEMATICS COURSE OBJECTIVES

Math 036a: Basic Math

- Evaluate expressions using the correct order of operations involving decimals, fractions, and integers without the use of a calculator.
- Solve real-world problems using and applying ratios, proportions, percentages, measurements, basic geometry, and statistics.
- Convert in and between the U.S. and Metric systems of measurements.

Math 038: Prealgebra

- Evaluate expressions using the correct order of operations involving fractions and integers without the use of a calculator.
- Apply the properties of operations, like terms, and integer exponents to simplify algebraic expressions and polynomials.
- Solve real-world problems using proportions, percentages, basic geometry, and multiple-step linear equations.
- Factor basic polynomials.
- Graph linear equations.

Math 040: Beginning Algebra

- Solve a simple linear equation or an equation involving parentheses.
- Add, subtract, multiply, and divide signed numbers.
- Graph a line given its equation.
- Write the equation of a line if given specific information about the line.
- Use exponent rules to simplify algebraic or numeric expressions.
- Factor various types of algebraic expressions.
- Simplify or perform operations involving rational expressions.
- Solve rational equations.
- Solve real-world application problems that involve beginning algebra.

Math 097: Math Literacy

- Perform basic operations with integers, fractions, and decimals.
- Use and interpret exponents, particularly involving powers of 10 and place values.
- Use percent to solve problems.
- Use ratios and proportions to solve problems.
- Perform conversions of common units of time, length, area, and volume.
- Use and interpret prefixes within the metric system.
- Perform conversion of units to and from the metric system.
- Recognize additive and multiplicative patterns and write these patterns algebraically.
- Evaluate algebraic formulas including those with multiple variables.
- Use calculators effectively, including scientific notation.
- Solve linear and quadratic equations.
- Graph linear, quadratic, and exponential equations.
- Solve application problems using linear and exponential equations.
- Use and interpret data sets, types of data, and displays of data.
- Find the mean, median, quartiles, mode(s), and weighted averages of a data set.

APPENDIX E: INSTITUTIONAL RESEARCH APPROVALS

FERRIS STATE UNIVERSITY

INSTITUTIONAL REVIEW BOARD
1010 Campus Drive FLITE 410 Big Rapids, MI 49307
www.ferris.edu/irb

Date: August 10, 2021

To: Susan DeCamillis, EdD and Jon Meeuwenberg
From: Gregory Wellman, R.Ph, Ph.D, IRB Chair
Re: IRB Application *IRB-FY20-21-221 Community College Developmental Math Number Sense*

The Ferris State University Institutional Review Board (IRB) has reviewed your application for using human subjects in the study, *Community College Developmental Math Number Sense (IRB-FY20-21-221)* and approved this project under Federal Regulations Expedited Review Approved 7. Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

Your protocol has been assigned project number IRB-FY20-21-221. Approval mandates that you follow all University policy and procedures, in addition to applicable governmental regulations. Approval applies only to the activities described in the protocol submission; should revisions need to be made, all materials must be approved by the IRB prior to initiation. In addition, the IRB must be made aware of any serious and unexpected and/or unanticipated adverse events as well as complaints and non-compliance issues.

This project has been granted a waiver of consent documentation; signatures of participants need not be collected. Although not documented, informed consent is a process beginning with a description of the study and participant rights, with the assurance of participant understanding. Informed consent must be provided, even when documentation is waived, and continue throughout the study.

As mandated by Title 45 Code of Federal Regulations, Part 46 (45 CFR 46) the IRB requires submission of annual status reports during the life of the research project and a Final Report Form upon study completion. Thank you for your compliance with these guidelines and best wishes for a successful research endeavor.

Regards,

Gregory Wellman, R.Ph, Ph.D, IRB Chair
Ferris State University Institutional Review Board

Muskegon Community College

221 South Quarterline ◆ Muskegon, MI 49442

July 20, 2021

TO: Institutional Review Board
Ferris State University
CC: Muskegon Community College Office of Institutional Research and Grants
RE: Research approval for JB Meeuwenberg

JB Meeuwenberg has asked to use data collected in developmental math classes at Muskegon Community College in the Fall 2021 semester for his dissertation research for Ferris State University. As our institution does not yet have an Institutional Review Board, I am granting permission for this study contingent upon the IRB approval from Ferris State University.

According to JB's proposal, he is seeking to understand the level of number sense of community college developmental math students, how their number sense may improve throughout a typical developmental math course, and if there is a relationship between their number sense and their success in that course. This study has two data collection periods—one for a pretest and one for a posttest. The first phase will occur during the first two weeks of the fall semester: 8/23/2021–9/3/2021. The second phase will occur during the last two weeks of each class section, prior to the final exam. Because different class sections vary in their number of weeks, the second phase will begin on 9/20/2021 and end on 12/3/2021.

As JB's proposal describes, it is expected the data will be stored securely and once all data is collected, including the course grades, any information that could identify a student will be removed from the database.

Please contact me if additional information is needed.

Sincerely,

Kelley Conrad, Ph.D
Vice President for Academic Affairs