# ALIGNMENT OF THE STANDARD LEVEL MATHEMATICS COURSE OF THE INTERNATIONAL BACCALAUREATE DIPLOMA PROGRAM TO THE COMMON CORE STATE STANDARDS

by

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ABSTRACT

All public high school students are required to meet current state high school content standards despite enrollment in special programs, such as the International Baccalaureate Diploma Program. Recently, the state of Michigan adopted the Common Core State Standards to replace the previous Mathematics and English Language Arts content expectations. Therefore, this capstone project aligns the course content from the Standard Level Mathematics course, as part of the International Baccalaureate Diploma Program, to the Common Core State Standards to ensure students enrolled in the course will meet the high school math content requirements. The alignment document produced in this project demonstrates the correlation between each objective in the Standard Level IB math course to the Common Core Content Standard, as well as the primary Mathematical Practice Standard. The use of curriculum mapping is also discussed and initial stages of the fluid document were created, followed by recommendations for continued mapping by future instructors of the course.

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# CHAPTER 1

## INTRODUCTION OF CURRICULUM ALIGNMENT

The following project is a capstone experience completed as part of the Masters of Education (M.Ed) in Curriculum and Instruction. The project is designed to cultivate credentials and expertise relevant to my occupational content area and integrates the experiences and content learned from my graduate work.

This chapter contains a summary of the project and relates its importance to my professional work. The benefits the project provides to the school and the program are significant.

The Common Core State Standards are a consistent set of K-12 expectations that define what all students are expected to know and be able to do in English Language Arts and Mathematics to lead to career and college readiness. The Common Core State Standards is not a curriculum, but serves as the framework for the local school districts and educators to develop their curriculum around. For example, districts can decide what curriculum is appropriate to achieve the standards while allowing educators to determine what lessons within the curriculum best meet the needs of their particular students.

Prior to development of the Common Core State Standards in June of 2010, each state had independent English Language Arts and Mathematics standards specific to their individual state. Unfortunately, these standards were extremely inconsistent and demonstrated varying degrees of rigor across the country. Therefore, comparisons across state lines were not reliable. As a result, the Common Core State Standards were designed to ensure that all students graduating from high school are prepared to enter college or the workforce with the skills needed to be successful despite the location of their elementary and secondary schooling. Even more, the Common Core State Standards are benchmarked to international standards to ensure students are competitive in the current global economy (CCSSI, 2012). Though it is not mandatory to adopt the Common Core State Standards, according to Michigan Department of Education (2013), forty-five states have currently embraced them. The target date for the new state assessments aligned to the Common Core in Michigan is projected for the 2014-15 school year.

At a public school in a metropolitan suburb of Detroit, Michigan, the math curriculum, excluding IB courses, is currently under revision to align with the Common Core State Standards. This district was participating in a countrywide development and pilot of mathematics units that align to the Common Core State Standards. Once the units are finalized, the district curriculum maps will be updated to reflect these revisions. Even more, this district has committed to using the Rubicon Atlas Curriculum Management software to support implementation of the Common Core State Standards. This software will assist teacher's management of unit-based curriculum, as well as identify repetitions and gaps in instruction, link standards to assessments allowing for standards-based reports to be generated, and enables teachers' collaboration and sharing across grades, schools, and districts.

The district began examining the International Baccalaureate (IB) Diploma program in 2007. The original proposal included a K-10 grade IB preparatory programs as well as the 11-12 grade diploma program. However, due to severe budget cuts in the district, only the high school diploma program was approved. Upon approval, the district's Board of Education mandated a committee be formed to outline the implementation including prospective location, enrollment processes, trimester scheduling, and a projected budget (Feasibility Study, 2010). After thorough research and presentation to the Board of Education, the high school was chosen to host the diploma program and was permitted to move forward with the application process to the International Baccalaureate Organization. Although trimester scheduling is not conducive to the demanding IB timeline, in the spring of 2013, the high school in the district was approved as an IB World School.

Students interested in participating in the IB Diploma Program at the high school level typically complete an interest form during their eighth grade year. If more students are interested than slots available, a lottery system is administered. Selected students are enrolled in IB-preparatory classes in their ninth and tenth grade years to prepare them for the expectations and rigor of the program beginning their 11$^{th}$ grade year. Upon completion of the 2-year program, students sit for an external assessment for each subject that is evaluated by an international team of IB moderators. However, there is an option to allow students to chose to take select IB classes to earn individual certificates in desired courses rather than the entire IB diploma. However, if the student wishes to earn the diploma, he or she must take one class from each of the six domains as well as the additional three core requirements later to be explained. Successful completion of the final external examination from all six domains, in addition to three core requirements, grants a student the International Baccalaureate Diploma along with the standard high school diploma.

## Statement of Need

Students enrolled in the International Baccalaureate Diploma Program are expected to sufficiently adhere to the current state standards as well as the International Baccalaureate standards. Therefore, teachers must prepare students for state assessments in addition to the external cumulative IB assessment. According to the Smarter Balanced Assessment Consortium (2012), the Smarter Balanced is one of two multistate consortia awarded funding from the U.S. Department of Education in 2010 to develop an assessment system aligned to the Common Core State Standards by the 2014-15 school year. Furthermore, the high school with the new IB diploma program will have their first class of students complete their first external IB examinations in the same 2014-15 school year. Therefore, it is essential to integrate the content from the IB courses to the Common Core State Standards to ensure all objectives are being addressed and students are prepared for both examinations. Currently, the high school does not have an updated curriculum map that directly aligns the IB objectives to the Common Core State Standards. This project addresses the alignment of only one of the IB math courses with the intent that the other courses will align in the future.

## Purpose of the Project

The purpose of the project is to develop a plan to implement the integration of the Common Core Stand Standards (CCSS) with the objectives in Standard Level Mathematics in the International Baccalaureate Diploma Program, while also linking units to the IB's learner profile and theory of knowledge requirements.

## Phase plan

The project is divided into three phases. Phase 1 is to develop a document for the administration and faculty to show the alignment of the CCSS and IB objectives in Standard Level Mathematics. The result of phase 1 will be a document that can be distributed to the administration and faculty. Phase 2 is to develop a Curriculum Map with the aligned content to the existing curriculum. This will be completed on the Atlas Rubicon Curriculum Management software system. The result of Phase 2 will be a curriculum map that contains the Common Core State Standards, the IB objectives, units in the course, and links to the IB Learner Profile and Theory of Knowledge. Finally, phase 3 will be to present the plan to the appropriate groups of individuals and teach additional staff members how to efficiently enter their IB curriculum into the Atlas Rubicon system. The result of Phase 3 will be ongoing dialogue and collaboration with staff regarding the importance of curriculum mapping, curriculum alignment, and the impact they have on student achievement.

# CHAPTER 2

# LITERATURE REVIEW

This chapter contains the background literature related to the project. It is organized around the following topic areas: (1) the Common Core State Standards Initiative; (2) Michigan Implementation of the Common Core State Standards; (3) the International Baccalaureate Diploma Program; (4) Curriculum Mapping, and (5) Curriculum Alignment.

## Common Core State Standards Initiative

According to the Michigan Department of Education (2013), the K-12 Common Core State Standards were developed through a state-led initiative coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). The group leading the initiative was comprised of parents, teachers, local administrators, content experts from across the country, business leaders, and state leaders (Common Core State Standards Initiative [CCSSI], 2012). In addition, the NGA center and CCSSO provided public comment periods for anyone to submit feedback on draft standards documents (CCSSI, 2012). Most importantly, teachers have had periods to provide input in the development of the standards since they are the experts who engage with the students on an everyday basis

(CCSSI, 2012). The National Education Association (NEA), American Federation of Teachers (AFT), National Council of Teachers of Mathematics (NCTM), and the National Council of Teachers of English (NCTE) are among some of the organizations that have been active in providing feedback on the draft standards (CCSSI, 2012).

The standards are voluntary for each state to adopt across the country. The goal of the standards is to ensure all students graduating from high school receive the same expectations and standards regardless of what state they live in. This is to ensure America retains its competitive edge and adequately prepares students for university studies or the workplace (CCSSI, 2012). The Common Core State Math Standards are broken up into two sets of standards: The Standards for Mathematical Practices and The Standards for Mathematical Content. The Mathematical Practices include a list of eight goals for all educators to develop in their students and appear at every grade level. The Mathematical Practices include: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure; and (8) Look for and express regularity in repeated reasoning. The standards for mathematical content are "organized into six Conceptual Categories, then into Domains (large groups that progress across grades) and finally by Clusters (groups of related standards similar to the Topics in the High School Content Expectations)." (Michigan Department of Education [MDE], 2001-20013).

## Michigan's Implementation of the CCSS

According to the Michigan Department of Education (2001-2013), Michigan began the draft stages of aligning to the Common Core State Standards in 2009.

Following the public release of the K-12 standards in March 2010, Michigan welcomed legislature leadership comments (MDE, 2001-2013). The state board approved the Common Core State Standards in June 2010, encouraging districts to begin implementation (MDE, 2001-2013). With the adoption of new standards, a new assessment was needed. Therefore, two voluntary consortia began developing common assessments that align to the Common Core State Standards. The first assessment is the Partnership for Assessment of Readiness for College and Career (PARCC). The second assessment is the Smarter Balanced Assessment Consortium (SBAC). Michigan has adopted the Smarter Balanced Assessment, which began piloting in various districts across the state in 2013. Full implementation of the new assessments are projected to begin in the 2014-15 school year (MDE, 2001-2013).

## The International Baccalaureate Program

According to the International Baccalaureate Organization (n.d.), the IB program was originally founded in Geneva, Switzerland in 1968 as a non-profit educational organization. It then gained popularity throughout Europe to serve the children of diplomats who often transferred districts throughout their schooling careers. The Diploma Program offers college preparatory work for students ranging from age 16 to 19 years old at a standardized and rigorous level that appealed to the transient families. Following the successes of the Diploma Program, in 1994 the organization developed the Middle Years Program (MYP) for students aged 11 to 16 years old. This program provides an introduction to the rigor expected at the secondary level while focusing on developing reflective and critical thinkers. The organization again expanded in 1997 to establish the Primary Years Program (PYP) for children aged 3 to 12 years old. The goal of the

Primary Years Program is to develop the whole child as an inquirer, both inside and outside of the classroom (IBO, n.d.). The three programs together provide a coherent sequence of education that develops the whole individual focusing on the intellectual, emotional, and social growth of the child.

The Diploma Program is comprised of six discipline areas, consisting of Language and Literature, Language Acquisition, Individuals and Society, Experimental Sciences, Mathematics, and the Arts. In addition, diploma students must also complete three compulsory activities: a Creativity Action Service (CAS) requirement, an extended research paper consisting of at least 4,000 words, and a Theory of Knowledge (TOK) class that unifies the academic courses and focuses on critical thinking skills and ways of "knowing". Every IB class is required to demonstrate TOK philosophies relating to their specific content throughout the duration of the course. Even more, the International Baccalaureate Diploma Program strives to foster 10 attributes considered to encompass an international learner ready for a global economy. These traits, known as the IB Learner Profile, are inquirers, knowledgeable, thinkers, communicators, principled, open-minded, caring, risk-takers, balanced, and reflective (IBO, n.d.). All activities within the IB curriculum should cultivate one or more of the mentioned Learner Profile characteristics, contributing to a well rounded internationally minded student prepared for a global post secondary education and career.

Students enroll in the IB courses their $11^{th}$ and $12^{th}$ grade year. At the conclusion of the two-year program, students must pass final examinations in all six disciplines as well as successfully complete the three additional requirements to receive their International Baccalaureate Diploma along with their High School Diploma. The

International Baccalaureate Diploma Program is a highly respectable program among some of the top leading universities. Obtaining the International Baccalaureate Diploma demonstrates a students' academic motivation, perseverance, and time management are far superior to those of their peers. Similar to Advanced Placement courses, an increasing number of universities across the country are accepting successful completion of IB external assessments as conversion to college credit.

## Curriculum Mapping

The concept of curriculum mapping was born in the late 1970s by Fenwick English (Burns, 2001). These early maps outlined what topics and skills were taught, in what order, and for how long. However, prior to the current technological advances, most of the data compilation relied on surveys, interviews, and manual data analyzing methods. Fortunately, data processing techniques brought curriculum mapping into the computer era (Burns, 2001). Today, curriculum mapping software programs exist, such as Atlas Rubicon, that provide districts with an user-friendly platform to organize, align, and disseminate information (Rubicon International, 2013). According to Heidi Jacobs (2009-20013) curriculum mapping requires teachers to reveal what they are actually doing in the classroom and share it electronically. In other words, a curriculum map is not a static document, but continually evolves as teachers update what they are truly teaching throughout the school year. This provides transparency and facilitates communication among other teachers and administrators, both within a school and with neighboring districts. According to Burns (2001), "curriculum mapping provides a comprehensive professional development tool for using data in instructional decision making, aligning curriculum, determining instruction, assessing with standards, and designing innovative and engaging classroom instruction" (Conclusion section, para. 4). Therefore, curriculum mapping is a "powerful tool that can transform low achieving schools into high-performing learning communities" (Burns, 2001, Conclusion section, para. 4).

According to Heidi Jacobs (2009-20013), curriculum mapping should be broken down into a four-phase development model. The first phase is laying the foundation. This allows the school to establish reasons for the mapping and develop an overall vision for the school. The second phase is launching the mapping initiative. During this phase, long-term support is established, individual maps are created, the review process is initiated, consensus maps are developed, and mapping strategies are mastered. The third phase involves maintaining, sustaining, and integrating the mapping process into the school. During this time, the process is embedded into the culture of the school. Even more, assessment data is merged into the maps, as well as literacy and other initiatives integrating into the plan. This is when a plan for implementation is created. Finally, the last phase is advanced mapping tasks. This phase involves continually updating the maps to reflect and document instruction that is actually taking place in the classroom. Using this plan of curriculum mapping allows educators to direct their instruction to ensure they are providing engaging activities that are aligned to state and national standards and assessments.

## Curriculum Alignment

Curriculum alignment is the actual process that links what is being taught in the classroom to the standards the students are responsible for. In other words, it ensures the assessments and objectives are addressed in the instructional process (Squires, 2012). According to a study by David Sqiures (2012), using Fenwick English's three category model of curriculum alignment, school districts can improve their student achievement. The three categories are the taught curriculum, the tested curriculum, and the written curriculum. The written curriculum is a document provided by the district on what content is to be taught in the course. The tested curriculum includes the standardized and state tests, as well as the summative assessments provided by the teacher. The taught curriculum comes from teachers enacting the written curriculum in the form of lessons, activities, lectures, etc. It is essential to ensure alignment among these categories. One way to achieve this is to align curriculum across courses (horizontally) and across grade levels (vertically). Once the curriculum is directly aligned to state and national standards, educators can identify gaps and overlaps. The goal is to ensure students are receiving the intended instruction and are given authentic assessments on the content they have been taught. Completing curriculum alignment brings us closer to this goal.

## Literature Review Summary

The Common Core State Standards and the International Baccalaureate both have similar philosophies of preparing students for post-secondary education and careers. Therefore, by developing the curriculum alignment of both entities and integrating the alignment into the foundation for building a curriculum map, the result will be a more cohesive and engaging educational experience for our 21st century learners.

# CHAPTER 3

# METHODOLOGY

The purpose of this chapter is to describe the methodology for each of the three phases of the project.

**Phase 1: Develop a document for the administration and faculty to show the alignment of the CCSS and IB objectives in Standard Level math.**

The International Baccalaureate Organization has completed numerous studies supporting the alignment of the CCSS with the IB objectives. Therefore, using their resources as a foundation, in conjunction with the curriculum I developed in 2011 for the Standard Level Mathematics course during the high school's application and authorization to become a World School, I aligned the Common Core Standards to the IB curriculum. *The Common Core: Clarifying Expectations for Teachers and Students – Math,* (Bainbridge, Holman, Baker, and Yuu, 2011) was my main resource for outlining the Common Core math content and mathematical practice standards. Therefore, by using this resource, I became more familiar with the new Common Core State Standards, which enabled me to link the standards to the IB objectives in the Standard Level math course. I collaborated with the IB Coordinator at Harrison High School for obtaining textbooks and IB resources as well as developed contacts in nearby districts with established IB diploma programs.

**Phase 2: Develop a Curriculum Map with the aligned content to the existing curriculum.**

During my IB training in the summer of 2010, I received materials to support the development of the curriculum for the Standard Level Mathematics course at my building. At that time, I outlined the sequence of objectives and formed units within the curriculum. However, the subject outline for the Standard Level math course has since been updated. Therefore, I obtained a copy of the IB textbook used at the high school and revised the curriculum previously created to more accurately reflect the updated IB math requirements. Since the high school obtained rights to the Rubicon Atlas curriculum mapping software, I completed my mapping work using this software platform. During this time, I integrated the alignment to the Common Core Standards within the curriculum map. I also provided links to the IB Learner Profile within each unit as well as Theory Of Knowledge applications suggested by the SL Math subject guide provided by the International Baccalaureate Organization. I utilized the International Baccalaureate Curriculum Center to explore Theory of Knowledge activities to integrate into the units and other professional organizations that provide engaging instructional activity suggestions.

**Phase 3: Present the plan to the appropriate group of individuals.**

After aligning the IB curriculum to the Common Core State Standards, I planed to present my results to the administrators, IB coordinator, and other IB math teachers at the high school to ensure we have a common vision for the program and facilitate the discussions to open communication. This corresponds to Heidi Jacob's (2009-20013) phase two of her recommended curriculum mapping implementation model. I planed to invite other staff members who will eventually be required to upload their curriculum to Atlas Rubicon. I was prepared to serve as a point person for the program to assist others, as well as receive feedback from the math department on the curriculum mapping for the Standard Level mathematics course that has been uploaded. It is crucial to highlight the benefits curriculum mapping and alignment will have on increasing student achievement to improve teacher buy in. Eventually, the district will progress to further phases of Jacob's curriculum mapping implementation model and teachers will begin to see the benefits the hard work and planning has on student achievement as curriculum mapping becomes imbedded into the school culture.

# CHAPTER 4

# PROJECT

## Curriculum Alignment

The first step in aligning the Standard Level Math course objectives to the Common Core Math Content Standards was to organize the IB curriculum into units, separated into groups of related objectives. The alignment document follows the International Baccalaureate subject guide syllabus but is left to the discretion of the teacher to rearrange the units or objectives to best fit the needs of the individual class. Therefore, the curriculum alignment document follows the IB order, while the curriculum map on Atlas Rubicon adjusts the unit placements, teaching the Unit 4 on vectors last.

The next step was to thoroughly familiarize oneself with the Math Content standards from the Common Core. *The Common Core: Clarifying Expectations for Teacher & Students – Math* (2011) was used extensively during the alignment process. Each applicable standard was linked directly to an IB unit and objective. The Common Core high school math standards are divided into 6 domains: modeling (*), number and quantity (NQ), algebra (A), functions (F), geometry (G), and statistics and probability (S). The abbreviations are represented by the first part of the content standard code. Within these domains, the standards are grouped into related items referred to as clusters.

The cluster titles are represented by the second section of the content standard code. Finally, the standard is given a number, not to be mistaken for a hierarchy order. The Common Core Math content standards contain additional standards intended for advanced mathematics courses. The advanced standards, though not required to be reached by all students, are indicated by (+) at the end of the content standard code. These advanced standards were the primary focus of the curriculum alignment since the Standard Level IB math course follows Algebra II and Pre-calculus.

*The Common Core: Clarifying Expectations for Teacher & Students – Math* (2011) not only organizes the high school math content standards, but also identifies the suggested Mathematical Practice standard that correlates to each content standard. Therefore, the alignment of practice standards to content standards was established by Bainbridge, et al. (2011).

The Common Core State Standards articulate what all students should know and be able to do when they graduate high school in order to be career and college ready. However, it is not expected that all students take calculus in high school. Nonetheless, the Standard Level IB Math course includes calculus content. Therefore, there are no Common Core State Standards that directly align to the calculus unit objectives. As a result, the calculus unit has been omitted from the alignment document. The following document aligns the Standard Level IB Math class to the Common Core State Standards:

# Standard Level IB Math alignment to the Common Core State Standards

| Topic/unit (as identified in the IB subject guide) | IB Content Objectives | Common Core State Standards | |
|---|---|---|---|
| | | Content Standards | Mathematical Practice Standards |
| **Unit 1 – Algebra** | | | |
| 1.1 | Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series.<br><br>Sigma notation.<br><br>Applications. | A.SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. | #8 Look for an express regularity in repeated reasoning. |
| | | F.IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers | #2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | F.BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | #4 Model with mathematics.<br><br>#7 Look for and make use of structure. |
| 1.2 | Elementary treatment of exponents and logarithms.<br><br>Laws of exponents; laws of logarithms.<br><br>Change of base. | N.RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | #3 Construct viable arguments and critique the reasoning of others.<br><br>#7 Look for and make use of structure. |

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| | | | #8 Look for and express regularity in repeated reasoning. |
| | | N.RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. | #7 Look for and make use of structure. |
| | | A.SSE.2. Use the structure of an expression to identify ways to rewrite it. | #7 Look for and make use of structure. |
| | | A.SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | #7 Look for and make use of structure. |
| | | A.SSE.3c. Use the properties of exponents to transform expressions for exponential functions. | #7 Look for and make use of structure. |
| 1.3 | The binomial theorem: expansion f $(a+b)^n$, $n \in \mathrm{N}$.<br><br>Calculation of binomial coefficients using Pascal's triangle and $\binom{n}{r}$. | A.APR.5 Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle. | #7 Look for and make use of structure. |
| **Unit 2 – Functions and Equations** | | | |
| 2.1 | Concept of function $f: x \mapsto f(x)$.<br><br>Domain, range; image (value).<br><br>Composite functions.<br><br>Identity function. Inverse function $f^{-1}$. | F.IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. | #6 Attend to precision.<br><br>#7 Look for and make use of structure. |
| | | F.IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. | #4 Model with mathematics. |
| | | F.BF.1b. Combine standard function types using arithmetic operations. | #4 Model with mathematics. |

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| | | | #7 Look for and make use of structure. |
| | | F.BF.1c. (+) Compose functions. | #4 Model with mathematics.<br><br>#7 Look for and make use of structure. |
| | | F.BF.4a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. | #7 Look for and make use of structure. |
| | | F.BF.4b. (+) Verify by composition that one function is the inverse of another. | #7 Look for and make use of structure. |
| | | F.BF.4c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. | #7 Look for and make use of structure. |
| 2.2 | The graph of a function; its equation $y = f(x)$.<br>Function graphing skills.<br><br>Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range.<br><br>Use of technology to graph a variety of functions, including ones not specifically mentioned.<br><br>The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$. | F.IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. | #1 Make sense of problems and persevere in solving them.<br><br>#7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | F.IF.7a. Graph linear and quadratic functions and show intercepts, maxima, and minima. | #7 Look for and make use of structure. |

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| | | | #8 Look for and express regularity in repeated reasoning. |
| | | F.IF.7b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | F.IF.7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | F.IF.7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| 2.3 | Transformations of graphs.<br><br>Translations: $y = f(x) + b$; $y = f(x - a)$.<br><br>Reflections (in both axes): $y = -f(x)$; $y = f(-x)$.<br><br>Vertical stretch with scale factor *p*: $y = pf(x)$.<br><br>Stretch in the *x*-direction with scale factor $\frac{1}{q}$: $y = f(qx)$. | F.BF.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. | #5 Use appropriate tools strategically.<br><br>#7 Look for and make use of structure. |

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| | Composite transformations. | | |
| 2.4 | The quadratic function $x \mapsto ax^2+bx+c$: its graph, $y$-intercept (0, $c$).<br><br>Axis of symmetry.<br><br>The form $x \mapsto a(x-p)(x-q)$, $x$-intercepts ($p$,0) and ($q$,0).<br><br>The form $x \mapsto a(x-h)^2+k$, vertex ($h$, $k$). | A.SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | #7 Look for and make use of structure. |
| | | A.SSE.3a. Factor a quadratic expression to reveal the zeros of the function it defines. | #7 Look for and make use of structure. |
| | | A.SSE.3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | #7 Look for and make use of structure. |
| | | F.IF.8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | #2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure. |
| 2.5 | The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature.<br><br>The rational function $x \mapsto \frac{ax+b}{cx+d}$ and its graph.<br><br>Vertical and horizontal asymptotes. | F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | F.IF.7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | F.IF.7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | #7 Look for and make use of structure. |

| | | | |
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| | | | #8 Look for and express regularity in repeated reasoning. |
| 2.6 | Exponential functions and their graphs: $x \mapsto a^x$, $a > 0$, $x \mapsto e^x$.<br><br>Logarithmic functions and their graphs: $x \mapsto \log_a x$, $x > 0$, $x \mapsto \ln x$, $x > 0$.<br><br>Relationships between these functions: $a^x = e^{x \ln a}$; $\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$. | F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | F.IF.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | F.IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | #2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure. |
| | | F.IF.8b. Use the properties of exponents to interpret expressions for exponential functions. | #2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure. |
| | | F.BF.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. | #3 construct viable arguments and critique the reasoning of others.<br>#6 Attend to precision. |
| | | F.LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | #2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure. |

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| | | | #8 Look for and express regularity in repeated reasoning. |
| | | F.LE.4. For exponential models, express as a logarithm the solution to ab?? = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. | #5 Use appropriate tools strategically.<br><br>#7 Look for and make use of structure. |
| 2.7 | Solving equations, both graphically and analytically.<br><br>Use of technology to solve a variety of equations, including those where there is no appropriate analytical approach.<br><br>Solving $ax^2 + bx + c = 0,\ a \neq 0$.<br><br>The quadratic formula.<br><br>The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.<br><br>Solving exponential equations. | N.CN.7. Solve quadratic equations with real coefficients that have complex solutions. | #1 Make sense of problems and persevere in solving them.<br><br>#7 Look for and make use of structure. |
| | | N.CN.8. (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | N.CN.9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | #7 Look for and make use of structure. |
| | | A.REI.4. Solve quadratic equations in one variable. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | A.REI.4a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |

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| | | A.REI.4b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| 2.8 | Applications of graphing skills and solving equations that relate to real-life situations. | N.Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | #2 Reason abstractly and quantitatively.<br><br>#3 construct viable arguments and critique the reasoning of others. |
| | | N.Q.2. Define appropriate quantities for the purpose of descriptive modeling. | #4 Model with mathematics. |
| | | N.Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | #5 Use appropriate tools strategically.<br><br>#6 Attend to precision. |
| | | F.BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | #4 Model with mathematics.<br><br>#7 Look for and make use of structure. |
| **Unit 3 – Circular Functions and Trigonometry** | | | |
| 3.1 | The circle: radian measure of angles; length of an arc; area of a sector. | F.TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | #6 Attend to precision. |
| | | F.TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to | #2 Reason abstractly and quantitatively. |

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| | | all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | #3 construct viable arguments and critique the reasoning of others.<br><br>#6 Attend to precision. |
| 3.2 | Definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle.<br><br>Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$.<br><br>Exact values of trigonometric rations of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples. | F.TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | #2 Reason abstractly and quantitatively.<br><br>#3 construct viable arguments and critique the reasoning of others.<br><br>#6 Attend to precision. |
| | | F.TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for p/3, p/4 and p/6, and use the unit circle to express the values of sine, cosines, and tangent for x, p + x, and 2p – x in terms of their values for x, where x is any real number. | #2 Reason abstractly and quantitatively.<br><br>#3 construct viable arguments and critique the reasoning of others.<br><br>#7 Look for and make use of structure. |
| 3.3 | The Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$.<br><br>Double angle identities for sine and cosine.<br><br>Relationship between trigonometric ratios. | F.TF.8. Prove the Pythagorean identity sin²(?) + cos²(?) = 1 and use it to calculate trigonometric ratios. | #7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | F.TF.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | #3 construct viable arguments and critique the reasoning of others.<br><br>#7 Look for and make use of structure. |

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| 3.4 | The circular functions $\sin x$, $\cos x$ and $\tan x$: their domains and ranges; amplitude, their periodic nature; and their graphs.<br><br>Composite functions of the form $f(x) = a\sin(b(x+c)) + d$.<br><br>Transformations.<br><br>Applications. | F.TF.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | #2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure.<br><br>#8 Look for and express regularity in repeated reasoning. |
| | | F.TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | #4 Model with mathematics.<br><br>#7 Look for and make use of structure. |
| | | N.Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | #2 Reason abstractly and quantitatively.<br><br>#3 construct viable arguments and critique the reasoning of others. |
| | | N.Q.2. Define appropriate quantities for the purpose of descriptive modeling. | #4 Model with mathematics. |
| | | N.Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | #5 Use appropriate tools strategically.<br><br>#6 Attend to precision. |
| 3.5 | Solving trigonometric equations in a finite interval, both graphically and analytically.<br><br>Equations leading to quadratic equations in $\sin x$, $\cos x$ or $\tan x$. | G.SRT.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems. | #1 Make sense of problems and persevere in solving them.<br><br>#2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure. |

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| 3.6 | Solution of triangles.<br><br>The cosine rule.<br><br>The sine rule, including the ambiguous case.<br><br>Area of a triangle, $\frac{1}{2}ab\sin C$.<br><br>Applications. | G.SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. | #1 Make sense of problems and persevere in solving them.<br><br>#4 Model with mathematics. |
| | | G.SRT.9. (+) Derive the formula A = ½ ab sin© for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | #2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure. |
| | | G.SRT.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems. | #1 Make sense of problems and persevere in solving them.<br><br>#2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure. |
| | | G.SRT.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | #1 Make sense of problems and persevere in solving them.<br><br>#4 Model with mathematics. |
| | | N.Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | #2 Reason abstractly and quantitatively.<br><br>#3 construct viable arguments and critique the reasoning of others. |
| | | N.Q.2. Define appropriate quantities for the purpose of descriptive modeling. | #4 Model with mathematics. |
| | | N.Q.3. Choose a level of accuracy appropriate to limitations on | #5 Use appropriate tools strategically. |

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| | | measurement when reporting quantities. | #6 Attend to precision. |
| | | **Unit 4 - Vectors** | |
| 4.1 | Vectors as displacements in the plane and in three dimensions.<br><br>Components of a vector; column representation $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$.<br><br>Algebraic and geometric approaches to the following:<br><br>- The sum and difference of two vectors; the zero vector, the vector $-\mathbf{v}$;<br><br>- Multiplication by a scalar, $k\mathbf{v}$; parallel vectors;<br><br>- magnitude of a vector $\lvert\mathbf{v}\rvert$;<br><br>- unit vectors; base vectors; **i**, **j**, and **k**;<br><br>- position vectors $\overrightarrow{OA} = \mathbf{a}$;<br><br>- $\overrightarrow{AB} = \overrightarrow{OB} = \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$ | N.VM.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. | #6 Attend to precision. |
| | | N.VM.2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. | #7 Look for and make use of structure. |
| | | N.VM.4. (+) Add and subtract vectors. | #1 Make sense of problems and persevere in solving them.<br><br>#2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure. |
| | | N.VM.4a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. | #1 Make sense of problems and persevere in solving them.<br><br>#2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure. |
| | | N.VM.4b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. | #1 Make sense of problems and persevere in solving them.<br><br>#2 Reason abstractly and quantitatively. |

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| | | | #7 Look for and make use of structure. |
| | | N.VM.4c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. | #1 Make sense of problems and persevere in solving them.<br><br>#2 Reason abstractly and quantitatively.<br><br>#7 Look for and make use of structure. |
| | | N.VM.5. (+) Multiply a vector by a scalar. | #7 Look for and make use of structure. |
| | | N.VM.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. | #7 Look for and make use of structure. |
| 4.2 | The scalar product of two vectors.<br><br>Perpendicular vectors; parallel vectors.<br><br>The angle between two vectors. | N.VM.5a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(v?, v?) = (cv?, cv?). | #7 Look for and make use of structure. |
| | | N.VM.5b. Compute the magnitude of a scalar multiple cv using \|\|cv\|\| = \|c\|v. Compute the direction of cv knowing that when \|c\|v ? 0, the direction of cv is either along v (for c > 0) or against v (for c < 0). | #7 Look for and make use of structure. |
| 4.3 | Vector equation of a line in two and three dimensions: $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.<br><br>The angle between two lines. | N.VM.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors. | #1 Make sense of problems and persevere in solving them.<br><br>#4 Model with mathematics. |

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| 4.4 | Distinguishing between coincident and parallel lines.<br><br>Finding the point of intersection of two lines.<br><br>Determining whether two lines intersect. | | |
| **Unit 5 – Statistics and Probability** | | | |
| 5.1 | Concepts of population, sample, random sample, discrete and continuous data.<br><br>Presentation of data: frequency distributions (tables); frequency histograms with equal class intervals.<br><br>Box-and-whisker plots; outliers.<br><br>Grouped data: use of mid-interval values for calculations; interval width; upper and lower interval boundaries; modal class. | S.ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots). | #1 Make sense of problems and persevere in solving them.<br><br>#5 Use appropriate tools strategically. |
| | | S.ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | #1 Make sense of problems and persevere in solving them.<br><br>#5 Use appropriate tools strategically. |
| | | S.ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | #1 Make sense of problems and persevere in solving them.<br><br>#5 Use appropriate tools strategically. |
| | | S.IC.1. Understand that statistics is a process for making inferences about population parameters based on a random sample from that population. | #2 Reason abstractly and quantitatively. |
| | | S.IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | #3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics.<br><br>#6 Attend to precision. |

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| 5.2 | Statistical measures and their interpretations.<br><br>Central tendency: mean, median, mode.<br><br>Quartiles, percentiles.<br><br>Dispersion: range, interquartile range, variance, standard deviation.<br><br>Effect of constant changes to the original data.<br><br>Applications. | S.ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | #1 Make sense of problems and persevere in solving them.<br><br>#5 Use appropriate tools strategically. |
| | | S.ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | #1 Make sense of problems and persevere in solving them.<br><br>#5 Use appropriate tools strategically. |
| | | S.ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve. | #2 Reason abstractly and quantitatively.<br><br>#5 Use appropriate tools strategically.<br><br>#7 Look for and make use of structure. |
| | | S.IC.5. Use data from a randomized experiment to compare two treatments; justify significant differences between parameters through the use of simulation models for random assignment. | #3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics.<br><br>#6 Attend to precision. |
| | | S.IC.6. Evaluate reports based on data. | #2 Reason abstractly and quantitatively.<br><br>#3 construct viable arguments and critique the reasoning of others. |
| 5.3 | Cumulative frequency; cumulative frequency graphs, use to find median, quartiles, percentiles. | S.ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, | #2 Reason abstractly and quantitatively. |

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| | | marginal and conditional relative frequencies). Recognize possible associations and trends in the data. | |
| 5.4 | Linear correlation of bivariate data.<br><br>Pearson's product-moment correlation coefficient *r*.<br><br>Scatter diagrams; lines of best fit.<br><br>Equation of the regression line<br>.<br>Scatter diagrams; lines of best fit.<br><br>Equation of the regression line of *y* on *x*.<br><br>Use of the equation for prediction purposes.<br><br>Mathematical and contextual interpretation. | S.ID.6. Represent data on two quantitative variables on a scatter plot and describe how the variables are related. | #2 Reason abstractly and quantitatively.<br><br>#4 Model with mathematics. |
| | | S.ID.6a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. | #2 Reason abstractly and quantitatively.<br><br>#4 Model with mathematics. |
| | | S.ID.6b. Informally assess the fit of a model function by plotting and analyzing residuals. | #2 Reason abstractly and quantitatively.<br><br>#4 Model with mathematics. |
| | | S.ID.6c. Fit a linear function for scatter plots that suggest a linear association. | #2 Reason abstractly and quantitatively.<br><br>#4 Model with mathematics. |
| | | S.ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear fit in the context of the data. | #2 Reason abstractly and quantitatively.<br><br>#4 Model with mathematics.<br><br>#5 Use appropriate tools strategically. |
| | | S.ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit. | #4 Model with mathematics.<br><br>#5 Use appropriate tools strategically. |
| | | S.ID.9. Distinguish between correlation and causation. | #2 Reason abstractly and quantitatively. |

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| | | | #3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics. |
| | | S.IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. | #1 Make sense of problems and persevere in solving them.<br><br>#3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics. |
| | | S.IC.6. Evaluate reports based on data. | #2 Reason abstractly and quantitatively.<br><br>#3 construct viable arguments and critique the reasoning of others. |
| 5.5 | Concepts of train, outcome, equally likely outcomes, sample space ($U$) and event.<br><br>The probability of an event $A$ is $P(A)=\frac{n(A)}{n(U)}$<br>.<br><br>The complimentary events $A$ and $A'$ (not $A$).<br><br>Use of Venn diagrams, tree diagrams and tables of outcomes. | S.IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. | #1 Make sense of problems and persevere in solving them.<br><br>#3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics. |
| | | S.CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. | #1 Make sense of problems and persevere in solving them.<br><br>#2 Reason abstractly and quantitatively.<br><br>#4 Model with mathematics. |

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| | | | #7 Look for and make use of structure. |
| 5.6 | Combined events, $P(A \cup B)$.<br><br>Mutually exclusive events: $P(A \cap B) = 0$.<br><br>Conditional probability; the definition $P(A\|B) = P(A) = P(A\|B')$.<br><br>Probabilities with and without replacement. | S.IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. | #1 Make sense of problems and persevere in solving them.<br><br>#3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics. |
| | | S.CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). | #1 Make sense of problems and persevere in solving them. |
| | | S.CP.2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | #1 Make sense of problems and persevere in solving them.<br><br>#3 construct viable arguments and critique the reasoning of others. |
| | | S.CP.3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. | #1 Make sense of problems and persevere in solving them.<br><br>#6 Attend to precision. |
| | | S.CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide | #1 Make sense of problems and persevere in solving them.<br><br>#2 Reason abstractly and quantitatively. |

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| | | if events are independent and to approximate conditional probabilities. | #4 Model with mathematics.<br><br>#7 Look for and make use of structure. |
| | | S.CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. | #1 Make sense of problems and persevere in solving them.<br><br>#4 Model with mathematics. |
| | | S.CP.6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model. | #1 Make sense of problems and persevere in solving them.<br><br>#7 Look for and make use of structure. |
| | | S.CP.7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. | #1 Make sense of problems and persevere in solving them.<br><br>#7 Look for and make use of structure. |
| | | S.CP.8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B\|A) = P(B)P(A\|B), and interpret the answer in terms of the model. | #1 Make sense of problems and persevere in solving them.<br><br>#7 Look for and make use of structure. |
| 5.7 | Concept of discrete random variables and their probability distributions.<br><br>Expected value (mean), E(*X*) for discrete data. Applications. | S.IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. | #1 Make sense of problems and persevere in solving them.<br><br>#3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics. |
| | | S.IC.6. Evaluate reports based on data. | #2 Reason abstractly and quantitatively. |

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| | | | #3 construct viable arguments and critique the reasoning of others. |
| | | S.CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. | #1 Make sense of problems and persevere in solving them.<br><br>#2 Reason abstractly and quantitatively.<br><br>#4 Model with mathematics.<br><br>#7 Look for and make use of structure. |
| | | S.MD.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. | #1 Make sense of problems and persevere in solving them. |
| | | S.MD.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. | #1 Make sense of problems and persevere in solving them.<br><br>#4 Model with mathematics. |
| | | S.MD.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. | #2 Reason abstractly and quantitatively.<br><br>#4 Model with mathematics. |
| | | S.MD.5. (+)Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. | #3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics. |

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| | | S.MD.5a. Find the expected payoff for a game of chance. | #3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics. |
| | | S.MD.5b. Evaluate and compare strategies on the basis of expected values. | #3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics. |
| | | S.MD.6. (+)Use probabilities to make fair decisions | #3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics.<br><br>#5 Use appropriate tools strategically. |
| | | S.MD.7.(+) Analyze decisions and strategies using probability concepts | #3 construct viable arguments and critique the reasoning of others.<br><br>#4 Model with mathematics.<br><br>#5 Use appropriate tools strategically. |
| 5.8 | Binomial distribution.<br><br>Mean and variance of the binomial distribution. | S.ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | #1 Make sense of problems and persevere in solving them.<br><br>#5 Use appropriate tools strategically. |
| | | S.ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a | #2 Reason abstractly and quantitatively.<br><br>#5 Use appropriate tools strategically. |

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| | | procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve. | #7 Look for and make use of structure. |
| 5.9 | Normal distribution and curves.<br><br>Standardization of normal variables ($z$-values, $z$-scores).<br><br>Properties of the normal distribution. | S.ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve. | #2 Reason abstractly and quantitatively.<br><br>#5 Use appropriate tools strategically.<br><br>#7 Look for and make use of structure. |
| | | S.IC.6. Evaluate reports based on data. | #2 Reason abstractly and quantitatively.<br><br>#3 construct viable arguments and critique the reasoning of others. |
| **Unit 6 - Calculus** | | | |
| 6.1 | Informal ideas of limit and convergence.<br><br>Limit notation.<br><br>Definition of derivative from first principles as $f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$.<br><br>Derivative interpreted as gradient function and as rate of change.<br><br>Tangents and normals, and their equations. | | |
| 6.2 | Derivative of $x^n$ $(n \in Q)$, $\sin x$, $\cos x$, $\tan x$, $e^x$, and $\ln x$.<br><br>Differentiation of a sum and a real multiple of | | |

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| | these functions.<br><br>The chain rule for composite functions.<br><br>The product and quotient rules.<br><br>The second derivative.<br><br>Extension to higher derivatives. | | |
| 6.3 | Local maximum and minimum points.<br><br>Testing for maximum or minimum.<br><br>Points of inflexion with zero and non-zero gradients.<br><br>Graphical behavior of functions, including the relationship between the graphs of $f, f'$ and $f''$.<br><br>Optimization.<br><br>Applications. | | |
| 6.4 | Indefinite integration as anti-differentiation.<br><br>Indefinite integral of $x^n$ $(n \in Q)$, $\sin x$, $\cos x$, $\frac{1}{x}$, and $e^x$.<br><br>The composites of any of these with the linear function $ax+b$.<br><br>Integration by inspection, or substitution of the form $\int f(g(x))g'(x)dx$. | | |
| 6.5 | Anti-differentiation with a boundary condition | | |

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| | to determine the constant term.<br><br>Definite integrals, both analytically and using technology.<br><br>Areas under curves (between the curve and the $x$-axis).<br><br>Areas between curves.<br><br>Volume of revolution about the $x$-axis. | | |
| 6.6 | Kinematic problems involving displacement $s$, velocity $v$ and acceleration $a$.<br>Total distance travelled. | | |

## Rubicon Atlas

Rubicon Atlas is the platform purchased by the district to store and display the curriculum maps created by the district and local Intermediate School District. The result is a personalized curriculum repository for working documents, with the intention that it will be continually updated by the instructor of the course.

The first step of the curriculum mapping was to enter the IB content objectives, separated by units, into the course on Rubicon Atlas. The International Baccalaureate Organization has predetermined minimum hours required for each unit in the Standard Level math course. Therefore, my next step in the curriculum mapping was to develop the unit planner. The district has designated Standard Level IB Math as a two-year course. Therefore, the unit planner demonstrates units 1, 2, 3, and 4 to be covered during the first year and units 5 and 6 to be covered the second year. The remaining time of year two should be used for the internal assessment and for review and preparation for the comprehensive external assessment. Pacing should be adjusted to meet the needs of the individual students in the course, holiday breaks, and trimesters the course meets. The unit planner exemplifies trimester 1 and 2 for both school years of the course.

Using the curriculum alignment document, Common Core State Standards were linked to each IB unit. By unwrapping the standards, teachers have a clear understanding of what students should know and be able to do in each unit. As a result, the big ideas and overarching questions for the unit emerge and are documented in the corresponding section of Rubicon Atlas. Once the teacher has articulated the essential questions to guide student understanding of the main ideas of each unit, formative and summative assessments can be developed. As these assessments are developed, they should be uploaded into Rubicon Atlas.

Upon uploading assessments into Rubicon Atlas, the program has provided a feature to link the content standards the user identified for the unit to each individual assessment. Therefore, the instructor can choose from a drop down menu on whether the content standard was introduced, practiced, or mastered on the given assessment. Using this feature, educators can analyze their courses and identify gaps and repetitions in the content standards while aligning both horizontally and vertically.

The last stage of developing the foundation for the curriculum map is to link the IB Learner Profile traits and Theory of Knowledge (TOK) connections to each unit in the course, as well as building the instructional activities and resources. Currently, the IB Learner Profile and TOK links are updated in Atlas Rubicon. The list of instructional strategies and activities are suggestions and may change with the development of the course as the teacher adjusts to their individual learners.

The curriculum map is intended to be a working document that educators use as a primary resource to manage the curriculum for the course. Rather than the archaic way of mapping the content, the sequence, and the timeline, often created in isolation from other teachers, contemporary curriculum mapping is a continual and transparent process.

# CHAPTER 5

# CONCLUSION

## Summary

Completing the alignment and curriculum map of the Standard Level Math course to the Common Core State Standards allowed me to unwrap the high school math standards and obtain a better understanding of what students should know and be able to do upon completion of the course. When the teacher has a more clear understanding of the goals, they become more transparent to the student. The Common Core State Standards outline what all students should know and be able to do to ensure career and college readiness. The International Baccalaureate Diploma program is a college preparatory program. Therefore, it is understandable these two frameworks would align nicely together. Now that there is alignment to a single math course in the high school track, the next step would be to vertically align the high school math course sequences to ensure a logical progression and that all common core state standards are met throughout the duration of a students' high school career.

I have learned that it is crucial for educators to unwrap standards and have a clear understanding of what students should know and be able to do for the course. The process of aligning the curriculum and continual curriculum mapping allows the content in class to be more transparent to not only other educators, but to the students. When both educators and students are clear of the content expectations, achievement should increase.

## Limitations

As described by Heidi Jacobs (2009-2013), curriculum mapping is not intended to be a static document, but rather an evolving document revealing the actual processes occurring in the classroom. Therefore, the product developed in Rubicon Atlas is not intended to be final, but used as a springboard for the next instructor of the Standard Level IB Math course. It is recommended the instructor use the material in Atlas Rubicon to develop units following the Understanding By Design approach. Once formative assessments, such as learning goals and scales are developed, they should be uploaded to the program for collaboration and feedback with other math instructors. This course curriculum map should grow and improve with time.

Heidi Jacobs' (2009-2013) implementation phases of curriculum mapping is recommended for the district to successfully integrate the Atlas Rubicon software into the school culture and equip teachers with the skills and resources needed to utilize the software to its full potential. One limitation of this project was the failure to perform phase 3 of presenting the alignment project and curriculum map to the high school faculty and staff. I was unable to present and become a resource for teachers regarding the program due to a relocation of assignment into another school district. However, the district has access to the curriculum map and alignment, which will support the next instructor for the Standard Level IB math course.

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