

THE IMPACT OF MATH ANXIETY AND OPTIMAL STRATEGIES FOR
UNDERSTANDING AND COPING WITH IT

by

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ABSTRACT

As our society becomes more technologically advanced, math understanding and aptitude continues to take on a more central role. Consequently, lack of math skills can diminish a student's future job prospects, and a number of students, for many different reasons, develop a self-identity as a poor math student that allows them to cope with a lack of math success. However, research has shown that as many as 80% of those students that believe they lack innate math competency actually, in fact, have reasonably adequate math skills, and are hindered more by their own belief in their mathematical inadequacy than by some fundamental math disability. By using empirically-tested general- and math-anxiety treatment methods, it is possible to help students treat this "math anxiety," and this is the foundation of the included math anxiety manual. The project will result in the creation of a manual that will offer students ways for understanding and coping with math anxiety.

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CHAPTER 1

INTRODUCTION

Introduction and Background

As a teacher at a small private boarding school that I will heretofore refer to as Woodbury, I have watched the development of an academic program that specializes in meeting the needs of students with learning differences. This specialization is in addition to its focus as a college preparatory school, so we emphasize differentiation and individualization as well as strength-based, hands-on education. We ensure that our academic curriculum meets or exceeds Michigan state and national public school standards, and teach our students how they can both learn that material using their strengths and advocate for assistance when their own understanding is not enough.

The school where I teach currently has about 50 enrolled students. The population is mostly (~80%) male, white, and midwestern American, though we also have students from across the country and from countries outside of the U.S. (China, India, and Angola). Because of the high cost of enrollment and the frequency of travel, our students come primarily from high-SES families.

As it is the target demographic of the school, most of our students come to us with learning deficits that precipitated their search for a boarding school. This means we teach

students with a variety of diagnoses. While ADD/ADHD is the most common, we also have students with a variety of other Learning Disorders (LDs), as well as a handful of students with high-functioning Autism (Autistic Spectrum Disorder, or ASD). The majority of our students (~85%) are also on some sort of medication for their diagnosed conditions.

In addition to the academic ramifications of their diagnosed learning disabilities, many of our students have difficulties with issues arising from the secondary social and emotional effects of these disabilities. For example, students with ADD often struggle reading social cues, (Kofler et al., 2011) and as a result they often develop asocial or antisocial tendencies to compensate for their lack of comfort with the nuances of interpersonal interactions. More pertinently, many students with a variety of learning disabilities may develop anxiety related to one or more school subjects (Nelson & Harwood, 2011). It is in this area that my interest lies.

Statement of Need

As a math and learning skills teacher at the school, I have spent the better part of 7 years now teaching math from Remedial Algebra all the way through AP Calculus. A great number of students that have come through my classes through the years that have told me how bad they were at math and how they have been worn down by years of confusion and failure. Students at our school come to us often having been beaten down by different factors in their little corners of the public school system. While they usually show that they are capable students in learning the material, they display many of the traits common to students with anxiety - such as restlessness, avoidance, and frustration in the classroom (Geist, 2010). These behaviors become barriers to their learning.

Students with avoidance behaviors will seek out other activities – such as going to the bathroom and/or trying to text their friends – rather than concentrating on the material being taught. Students that feel restless can either engage in movements that can be a distraction for others or attempt to avoid those movements and focus on not moving rather than on the material being taught. Students that experience frustration rather than see difficulties as a challenge are less likely to work hard to solve difficult material. As Geist (2010) points out, math anxiety is a significant barrier to learning, and being effective teachers requires that we tackle the problem head-on.

Woodbury's Math Program

Woodbury offers math courses very similar to the standard high school math curriculum - Algebra 1, Geometry, Algebra 2 (next year to be altered and retitled “Math Modeling”), Precalculus, and AP Calculus. Depending on enrollment and interest, we have offered other classes such as AP Statistics and Financial Literacy, but the focus remains on that core progression. Because we are a small school, most of our math classes heavily involve small-group or individual work with some in-class direct teacher instruction. While some students do go ahead in the math curriculum, the pacing for the average student tends to be slower than in public schools, because we individualize, assess, and reteach. Rather than teaching the curriculum and rewarding those who succeed on the first try, we try to use a variety of teaching methods to teach the students and understand that students will learn at different rates.

Performance of students on standardized math exams

There is a great deal of debate about the usefulness of standardized tests. At Woodbury, because there is a high rate of math anxiety in our school, we have found that

formal standardized tests are typically lacking in their ability to accurately assess student performance. While research on the effect of math anxiety on standardized testing is limited (Ryan, Ryan, Arbuthnot, & Samuels, 2007), the idea that math anxiety can affect standardized test performance is backed up by research that has shown that self-perception can have an effect on standardized-test performance (Haladyna & Downing, 2004). Since students with math anxiety typically have a lower self-perception of their math abilities than other students (Hembree, 1990), it would stand to reason that math anxiety would lead to lower performance on standardized tests. We therefore prefer consistent face-to-face interaction as a method of learning about students' true capabilities. We include non-standardized formalized tests as one of our measurement tools, but do not use them as the sole judge of student performance. Student experience with these tests is used primarily as a way of preparing them for either the college math experience or standardized tests such as the ACT or SAT (our school's scores on these tests are about at the state average, having seen improvement in recent years). A great deal of time is devoted to convincing students that they are not as bad at math as they think they are. The nature of the standardized test is that they allow students to "give up" on difficult questions and do not encourage the student to problem solve when they reach challenging questions. Our philosophy is that we need to encourage students to work through their difficulties and that all students are capable of doing more than they evidence on many of these tests.

Why did I choose this?

When we are successful at convincing students that they are not as bad at math as they think, it can make a huge difference. Many of my students become better math

students, and the joy I've found in helping them with that is such that I want to do it much more frequently.

As an example, at the start of summer school at the end of my first year, I had a female student **enroll** that (like many others) told me that she was bad at math and that past teachers had told her that "math was not her bag"; she had had a history of D's and F's in math, and had become very frustrated with her own struggles. When I first started working with her I could see that she was a capable math student, but she would quickly become very frustrated when doing the problems. It quickly became apparent that when I explained something to her and she did it correctly, upon arriving at the answer she would often say "but that's stupid" in frustration. About three days in, after she finished one of the problems that I was watching her do, she said "now what?" After a shrug I said "that's it, you got it." She asked "that's it?" I nodded, but she blurted "that's stupid" again. "Good, that means you understand it." I pointed out how she always said that when she got the right answer, but was uncomfortable with such an abstract answer being the correct one. In other words, she knew how to do the math, but her anxiety was getting in the way of her accepting that. At that point it was as if a light went off in her head, because she laughed and said "I say that all the time." I shrugged and said "then you are probably doing a lot better than you're letting yourself realize."

She went on to ace that class, aced precalculus in the fall, and is now working her way through law school. She really did get math, but never had someone show her that before. It stands to this day as one of the proudest moments of my teaching career - one of the few life-altering moments that we get as a teacher. In that rare instant, I smashed a giant wrecking ball into her wall of anxiety, and while she still remained anxious, it freed

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her up to do her math as she was capable.

Purpose of Project

The purpose of this project is to standardize a method of helping students with math anxiety in Woodbury. In this project I will develop an informational handbook that will help students, parents, and teachers at Woodbury mitigate the impact of math anxiety.

Statement of Project Goals

Goal 1: Describe the characteristics and symptoms of math anxiety that students have in instructional settings.

Goal 2: Define personal strategies, adjustments and treatments that math-anxious students can use to begin treatment on their own.

Goal 3: Identify common deficiencies in math-anxious students that those students can remediate while practicing anxiety-reduction techniques.

Benefit to Woodbury

Other teachers should have the experience of breaking down that wall of anxiety. Students should be able to feel comfortable with math in a way that they haven't been in a long time. Parents should be as proud of their children as that girl's parents were proud of her, and to better be able to help their children learn math, perhaps in a way they never could. Math anxiety is a condition that has had a negative impact on a great number of people and society in general, and I want to share what I know and help others that simply haven't had access to what I've learned over the years.

At our school specifically, I believe this handbook will be beneficial to not only our students, but their parents and our faculty, as well. The anxiety reduction techniques that the book will teach can likely be effectively applied in other areas besides math, and that

can only help our students, who have often developed anxiety associated with the struggles that they have had with school in the past. My hope is that this work can lead to significant changes in not only the performance of our students on standardized tests, but in their potential for success in college.

CHAPTER 2

LITERATURE REVIEW

Math Anxiety

Math proficiency is widely regarded by many researchers as an important skill and educational foundation in today's high-tech society (Ashcraft & Krause, 2007). Not only is it vital for success in a great number of high-paying jobs, (Arnold, Fisher, Doctoroff & Dobbs, 2002) but it also impacts a person's ability to stay on top of their own finances, which contributes to overall stress level and happiness (Krause, Newsom, & Rook, 2008). Unfortunately, while we live in an era where society has become unquestionably more technologically advanced and the number of math-related occupations has skyrocketed, we are failing to meet these demands for math-capable individuals, especially engineers (Driscoll, 2006). Clearly, math is an important area of study that too few people are going into. The question, then, is why? Are we as a society "bad at math," or is there another issue - one based less on ability and more on affinity and preference? There are many potential culprits, but this paper will focus on two: math disability and perception of math disability or a tendency to avoid it. This avoidance, which manifests itself in many different ways, may be due to something called math anxiety.

The concept of math anxiety has been studied since it was first noted as potentially different from general anxiety in the mid-70's (Hembree, 1990). Areas of study on the

phenomenon include topics such as causes (Jansen, et al., 2013; Mattarella-Micke, Mateo, Kozak, Foster, & Beilock, 2011), prevention (Furner & Duffy, 2002), impact (Ashcraft & Moore, 2009), and treatment (Hembree, 1990; Zettle, 2003), and the increasing body of research has contributed to a better understanding of the underlying causes.

Math anxiety is defined as involving “feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations.” (Richardson & Suinn, 1972, p. 551) Much like general or trait anxiety interfered with performance, it was postulated that “math anxiety” would interfere with capable students’ ability to demonstrate that performance on an exam. A great deal of research went into verifying the plausibility of a math anxiety that was a) separate in outcome from poor math performance, and b) not itself caused by the poor math performance. In a landmark meta-analysis in 1990, researcher Ray Hembree examined 151 studies on math anxiety to that point and concluded that there was a great deal of empirical evidence demonstrating that performance effects associated with math anxiety were only mildly correlated to math disabilities and general, trait, state, and test anxieties. The obvious conclusion was that math anxiety was a real condition with an underlying cause separate from both whatever caused other types of anxiety and whatever caused math disability itself. (Hembree, 1990)

Recently, much has been written about the United States’ perceived inability to compete globally in science and mathematics, and the inference (based on the purpose of the tests) is that low math test scores indicate low math ability. The drive for fixing that

problem, then, has led to more thorough high school math programs and more high-stakes testing as a way to promote accountability (Braun, 2004). This, in turn, has led to a higher-pressure environment for assessing core mathematics skills (Braun, 2004). Since math anxiety results in a tendency to do poorly compared to ability in high-pressure situations, this has resulted in a system that punishes students that have math anxiety disproportionately to their math ability level. It logically follows that remedying this situation will involve methods for reducing math anxiety, and that helping the portion of math-anxious students that do not succeed in these situations can help our society develop the skilled and educated worker base that it needs going forward.

Treatment Techniques

Unfortunately, math anxiety typically sets in at an early age - usually around middle school but as early as kindergarten (Arnold, et. al, 2002). Thankfully, while efforts to remediate math anxiety in the classroom are often limited by the time available for individual attention, there are individual remedies for math anxiety even after onset and things that parents and teachers can do to prevent the onset of math anxiety in the first place (Hembree, 1990). It's been found that although full-group or curricular interventions post-onset were mostly unsuccessful, out-of-class interventions such as cognitive behavioral therapy, cognitive restructuring, and systematic desensitization (SD, or "flooding") were "highly successful in reducing math anxiety levels." (Hembree, 1990, p.43).

Systematic Desensitization

Although its use has declined in recent years (McGlynn, Smitherman, & Gothard, 2004), Systematic Desensitization remains one of the most effective methods of treatment

for phobias and anxiety (McGlynn, Smitherman, & Gothard). It is a three-step process, defined by Wolpe (1958). The first step is to help the subject identify a “hierarchy of fear.” In doing this, the subject ranks his or her anxieties, starting with the least and ending with the greatest. The second step is to learn a strategy that can be used to cope with the anxiety when it occurs; muscle relaxation is a commonly-used one (Wolpe, 1958). The third step involves counter conditioning (Wolpe, 1958), a process whereby the subject is told to practice their coping strategies into practice by imagining the anxiety-producing situation while they practice the coping mechanism. When the stressor fails to evoke anxiety, the subject moves on to the next step on their hierarchy of fear and continues to practice the counter conditioning. By the end of the process, the subject should feel reduced levels of anxiety toward the source of their fears.

With math anxiety, the treatment process involves teaching the math-anxious student several calming methods (deep/slow breathing, muscle relaxation, etc.) and practicing them in isolation (in other words, while reading the book). When the student feels ready, a simple math problem is given - one that the student knows how to do, that is untimed, with no anxiety-producing stimuli. As the student becomes comfortable with repeated low-level exposures, the difficulty of the math is slowly increased, until the student is doing ability-appropriate math and training him/herself to respond using the calming techniques. Hembree (1990) notes that this technique is highly effective in treatment for math anxiety and follow-up analyses by other researchers over the years (e.g. Berman & Furner, 2003) have verified this technique as the most effective one available. Because of its proven effectiveness in treating math anxiety, this is the technique that I will be opting to build my manual around.

Mindfulness

Another philosophy that I will be incorporating into my manual will be Mindfulness, which is the belief that “experiencing the present moment nonjudgmentally and openly can effectively counter the effects of stressors.”(Hoffman, Sawyer, Witt, & Oh, 2010, p. 169). Mindfulness training involves learning techniques to increase a person’s capacity for “nonjudgmental observation and acceptance of bodily sensations, cognitions, emotional states, urges, and environmental stimuli as they arise” (Baer, Carmody, & Hunsinger, 2012, p.755). Because this has been shown to be an effective method for treating general anxiety disorders (Treanor, 2011), and because of the similar levels of effectiveness of general anxiety reduction therapies in treating math anxiety (Hembree, 1990), it seems reasonable to expect similar effectiveness with Mindfulness on treating math anxiety. Though research on the efficacy of mindfulness techniques on math anxiety is limited, several recent studies have found evidence that “Longer-term mindfulness practices might prove effective” (Brunye, et al., 2013, p. 5).

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CHAPTER 3

METHODOLOGY

Goal 1: Describe the characteristics and symptoms of math anxiety that students have in instructional settings

Being that math anxiety is a 40-year-old topic of research, by now there is a great deal of scholarly literature available about math anxiety and tests for it, although the ambiguity arising from the lack of quantitative measures of the disorder will have to be discussed in the final manual. The first step, then, will involve explaining the criteria in as clear and concise a way as possible for the reader, at a reading level appropriate for high school students potentially struggling with math anxiety. Delivering this information to students in a style to which they can relate will be an important step.

Goal 2: Define personal strategies, adjustments and treatments that math-anxious students can use to begin treatment on their own.

According to Hembree (1990), very little can be done at the whole-class level to remediate math anxiety once it has set in. That does not mean that students should be completely on their own, however, as the goal of a good teacher should be to educate their student to the best of their ability regardless of any deficits the student might have. By creating a readable manual with individually-focused coping skills and learning

strategies, I hope to provide a resource that students can use on their own to supplement whole-class instruction.

In doing this, I will include methods that students can use to begin compensating for any anxiety that they might be feeling toward math. Additionally, I will allocate space in the manual to re-teaching some of what I have found to be the most common curricular deficits that students have – such as fractions and order of operations - as both a way to teach potential math gaps and scaffold their anxiety-reduction development.

Goal 3: Identify common deficiencies in math-anxious students that those students can remediate while practicing anxiety-reduction techniques.

In order to properly use Systematic Desensitization strategies, it is imperative that the student feel comfortable with the type of mathematics being performed. Initial interventions using higher-level maths with which the student is struggling will likely be less successful than less-stressful math that the student is more capable of doing. Therefore, devising effective interventions will involve finding low-intensity math that is simple enough for the student to be able to do, interesting enough to hold the student's attention through the process, and challenging enough to be able to scaffold the student through the increasing levels of math. Since many math-anxious students may actually possess deficiencies of knowledge (compared to deficiencies of ability), chapters on more elementary topics, explained in ways that students may be unfamiliar with, may benefit students whether they already understand the discussed material or not.

CHAPTER 4

PROJECT RESULTS

This chapter contains a description of the process that I used to create the handbook. The goal of the project was to create and develop an informational handbook that will help students, parents, and teachers at Woodbury mitigate the impact of math anxiety. In order to develop the handbook, it was important to clearly define the characteristics and symptoms of math anxiety so that students, parents and teachers will recognize the symptoms of math anxiety.

Therefore, the manual is broken into two sections—Section 1 is intended to define and explain math anxiety while Section 2 contains a tutorial on oft-misunderstood math concepts that will allow students the opportunity to begin to try the anxiety-reduction techniques while at the same time teaching them those math skills that they may lack. The actual handbook may be found in Appendix A of this report.

Goal 1: Describe the characteristics and symptoms of math anxiety that students have in instructional settings.

The content from Goal 1 is the foundation for the first chapter in Section 1 of the handbook. The first two chapters Section 1 of the manual were devised around the results of a comprehensive literature search on the phenomenon math anxiety. The first chapter

of Section 1 is a fictional story of a student with many of the common symptoms and the environmental causes that contributed to her developing math anxiety. While the etiology of anxiety disorders is still a contested topic (Rapee, Schniering, & Hudson, 2009; Mineka & Zinbarg, 2006), I decided that a story of someone that went through many of the common classroom stressors would at least allow the reader to relate to someone whose school development is fairly typical for a student that develops math anxiety. Regardless of which psychological model that underlines the cause, most researchers believe its etiology and exacerbation is environmental in nature (Rapee et. al, 2009). Demonstrating to students, therefore, that even though the original causes may have been environmental, those stressors are no longer presents, and that means that the only continuing enforcement of that anxiety is the student him or herself. Getting the student to recognize that the locus of control over their math anxiety is internal should have a positive impact on their potential for improvement.

The second chapter of Section 1 is less subtle in purpose and more straightforward in its presentation of the symptoms of math anxiety. The goal, again, was to describe for the reader a variety of common symptoms that math anxiety may produce. In presenting a diversity of physical and psychological reactions, my intention was to get the reader to start to look at their own emotions and behavior in response to math and/or the anticipation of doing math. The manual included examples of physical (knee bouncing, increased heart rate, sweating, etc.), emotional (anger, frustration, depression, fear, apathy, etc.), and behavioral responses (leaving the classroom, not doing the work, etc.), and reiterated the idea that these are not necessarily natural behaviors, but are instead associated with the desire to avoid math altogether. Again, it's about establishing

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that internal locus of control - when students are able to feel that they can affect changes in their attitude toward something, they are more likely to actually affect those changes.

Goal 2: Define personal strategies, adjustments and treatments that math-anxious students can use to begin treatment on their own.

The third chapter of Section 1 of the manual deals with treatment suggestions and techniques. This chapter was developed around the results of an extensive literature search regarding treatment methods for people with math anxiety. While there are multiple different methods of treatment, the literature is clear that Systematic Desensitization is one of the most heavily studied and effective methods for treating math anxiety and that Mindfulness is one of the most heavily studied and effective methods for treating general anxiety, even if its use on math anxiety hasn't been validated. The difficulty was in presenting Systematic Desensitization and Mindfulness in an open and honest way. It's helpful to have them seem unusual and therefore intimidating, and make light of their quirkiness while at the same time making the reader both logically accept that they are effective treatments and worth attempting.

In my experience, teenagers can be a skeptical bunch, and are more self-conscious about trying things that are "weird" or would make them stand apart from what they perceive from their friends as normal behavior. The difficulty with both of those treatments, of course, is that they can require long-term commitment and improvement may only reveal itself after gradual improvement over a period of time, rather than offer a quick fix. For a written piece of work, requiring students to read and reread the same piece would be too much of a barrier to be effective. It became apparent that the manual would have to be somewhat redundant in order to maximize its effectiveness. Since the

manual is a lengthy work as it is, it made it easy for me to continually hit these core concepts without them feeling too repetitive; since students wouldn't be reading it all in one setting, I could go over SD, Mindfulness, and the symptoms of anxiety many times as long as I spread that out over many pages and altered how I presented them.

Goal 3: Identify common deficiencies in math-anxious students that those students can remediate while practicing anxiety-reduction techniques.

The content for this goal is found in Section 2 of the manual. Since the goal was to develop strategies for anxiety improvement and remediation, the bulk of the manual was in the section that claimed to reteach core deficiencies. The reason that it ended up this way was that this afforded me a better opportunity to reinforce these main math anxiety concepts, point out possible responses the reader may have to their own anxiety, and to give the opportunity for the reader to practice anxiety-reduction techniques in a safe and stress-free environment.

While I could find no specific literature asserting a set of common deficiencies, some research on important math skills (Wilson, 2009) and my own professional experience led me to the identification of the following eight:

1. basic math facts (addition, subtraction, multiplication, and division),
2. misconceptions about the nature of the equals sign,
3. fractions,
4. negative numbers,
5. parentheses,
6. order of operations,
7. variables, and

8. story problems.

These eight concepts are presented in the handbook in order of complexity for the purpose of sequencing content and avoid duplication of content. After beginning the work on the handbook using these eight concepts as a basis, it became clear that the last four would be best combined into only two chapters. This gave me six chapters worth of material. This format allowed me to explore the material in unusual ways in order to maintain interest, allow for me to highlight potential anxious responses from the reader, and provide concrete examples of how fundamental math concepts may simply require alternative methods of instruction to learn.

CHAPTER 5

REFLECTIONS, CONCLUSIONS, AND RECOMMENDATIONS

Math educators fight a daily battle with students that seem to not care about or have an active disdain for their math education. Given that many of our students have a long history of struggles with their previous schools, Woodbury is no different. The research shows that math anxiety, and not math disability, may be the root cause of a majority of these cases of environmentally-influenced student failures, and that those apathetic and aversive student behaviors are often a result of the students' anxious reactions to those math failures. Thankfully, anxiety has been shown to be remediable, and the hope is that with a focus on new techniques, our students will benefit from knowing both that their teachers believe that they can improve and that the students themselves have more of an impact on their own "math disorder" than they may have realized.

As far as the "therapeutic suggestions" go, I believe that the combination of Mindfulness techniques and Systematic Desensitization represents a good one-two punch that will both offer the appearance of a variety of treatments as well as present research-based, proven-effective systems that can be explained and scaffolded through writing, given that direct interaction isn't necessarily feasible for all of our students. The potential

to see the effectiveness of this in practice is exciting, and I have no reason to suspect that this won't work.

Structurally, I'm happy with the manual. The final layout of the parts and chapters and with the division of concepts flows and makes sense. It was easy to predict that the bulk of the text would be in Section 2, and that came to be. Because that section, for all intents and purposes, is optional, I felt that spending more time on this could reinforce the Teach-Practice-Apply model of math instruction without the pace of delivery being overwhelming to the reader, who is very possibly struggling with math anxiety that doesn't need any more help controlling them. As a corollary, there was a consideration for the importance of balancing the need for continual representation of the anti-anxiety strategies with the need to not overwhelm the student with too much information overall. The resulting balance involved focusing on the math anxiety first in the smaller section, then spiraling back through those concepts in Section 2.

From an aesthetic standpoint, I focused on making the language accessible to the manual's target audience (teenagers), and feel that, for the most part, this worked. Formatting mathematical writing can be really difficult, and I tried to make sure that there was enough white space to give equations room to breathe so that the reader isn't looking at an intimidating wall of letters and numbers. Additionally, while it was a chore to make sure that any explanations surrounding those equations was accurate and concise, yet readable at their level, I felt that, for the most part, I did this well.

There are two aspects of the manual that could use improvement. The first is the visual presentation – while the writing itself is readable and accessible, it is usually just a wall of text. If I were to publish this, I would put more emphasis on page design and

layout, with grids and colored text boxes, etc. - superfluous additions that would have added little substantively but increased the appeal of the text.

Secondly, while the anxiety-specific portion of the manual isn't all that long (roughly 19 pages), the manual's overall size may be daunting enough to trigger anxiety all by itself. Again, design considerations might help stem this - such as dividing Section 2 into detachable sections or making it more colorful. Editing for content would be a possibility, but when one considers that the book tries to re-teach many years' worth of fundamental math concepts, perhaps it has been pared down to the edge of effectiveness already and that improvement in this area could only come at the expense of the breadth of the text.

In the end, I feel that this manual can be an important addition to our school. As math department head, I will be encouraging our teachers to have copies on hand to give to students that are struggling, and as a learning skills teacher I will dole out a few of these to students for whom I think the manual can help. The research I did for this paper taught me a lot about the disorder in the process, and I am optimistic that this manual can have a positive impact not only on the results of our math curriculum, but on the lives our students beyond their tenure at Woodbury.

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APPENDIX A

MATH ANXIETY MANUAL

**Treating Math Anxiety
Or: How to Learn to
Stop Worrying
and Love Math**

Prologue: About Claire

Claire was an eighth-grade student that had struggled with math for years. She liked it when she was younger, when it was counting apples and money. It started getting hard, though, in third grade when she had to do those times tables tests. She wanted to be the best, but she knew that she wasn't finishing faster than most of her classmates. She also had a "strict" (mean) teacher that year that kept her in for recess to work on her math facts. She eventually got them, but she missed a lot of time with her friends, and it all stopped being fun for her. It wasn't fair, Claire thought, but her parents convinced her that even if she wasn't good at math, she needed the extra time to learn it.

Then the next year she got to fractions, and everything fell apart. She hated them from the first day - they had that little number bar and they didn't add right. She could never remember when to multiply and what to add, and she kept making simple mistakes that irritated her teacher, who was actually kind of a nice guy even though he made her do math by hand (why couldn't she just use a calculator?!) and made her stay inside from recess sometimes.

So when she got to math class in sixth grade and the teacher started using "x" instead of numbers, that's when Claire gave up. She wasn't going to be any good at it anyway, so why bother? She managed to work hard enough to pass, and by the time she was in eighth grade and taking pre-algebra, all she wanted to do every day was just get through the class period. She started skipping class on occasion, sometimes to go to the bathroom or sometimes to make an "emergency phone call," and sometimes on long walks to the school nurse. She was starting to fail, and knew that she needed to work harder, but she was bad at math and didn't care.

Then, when she found out that she was getting a referral to a special ed teacher, she was horrified. She wasn't like those kids! She wasn't dumb and was doing okay when she tried and could work harder and really wanted to stay in the same class as her friends. She argued about it with her parents and her eighth-grade teacher, but in the end she had to have someone come in and sit with her during her math class or something. It was so embarrassing that she wanted to cry.

The first day the helper came, Claire didn't even want to meet her. She didn't even want to be there, but she made the mistake of blabbing that one to her parents and they threatened to ground her if she wasn't in class. The helper - a Mrs. Astor - introduced herself and seemed nice, but then she sat next to her and Claire could see all of the kids in the class watching her and she wanted to just about die. Class started, and Claire didn't have her stuff out, Mrs. Astor actually told Claire to take out her book and notebook. Claire sighed angrily, rolled her eyes, ripped them out of her bag, slammed them on the desk and glared - she didn't want anyone to think that she had *asked* for the help. Claire was totally ready for when the other kids were going to ask her about it, too - "I don't *want* help, but my stupid parents are making that lady come

and just *sit* there with me like she cares. Whatever.”

After about ten or fifteen minutes of sitting there listening to her teacher *not shut up*, Claire got up to go to the bathroom. She was gone for about twenty minutes, and when she came back, the class was working on some problems that the teacher had given them. Claire sat down and Mrs. Astor leaned in.

“Do you want me to explain how this works,” she asked.

Claire shook her head and grabbed her pencil. She really didn’t want to get started. She instinctively reached for the cell phone in her pocket to text her friend, but thought better of it when she looked at Mrs. Astor out of the corner of her eye. She looked at the first problem, read it a little bit, and didn’t understand what it was asking. No big deal, she’d find out from one of her friends later (no way was she going to ask for help on the first day). With any luck, one of them might already have all of the answers, and then she could just copy those.

“So... three over x equals 6 over 8... what is the first step,” Mrs. Astor prompted.

Claire sighed and glanced at the problem. She shrugged her shoulders. This was embarrassing; she didn’t have to look, she *knew* her friends were all staring at her.

Mrs. Astor pointed at the paper and, politely, asked “what’s the problem asking you to find?”

Claire sighed impatiently. “Mrs. Astor, do we have to do this right now?” The lady nodded. “It’s just that I’ve had a headache all day and it’s been kinda hard to concentrate, and class is almost over...”

“There’s still 15 minutes left, Claire. We can get some of these done now so you can work on them later.”

“Yeah, but I don’t feel good, and I can just ask Laura later.”

Mrs. Astor sniffed. The lady cocked her head slightly and just looked at her, didn’t smile or frown or anything, and Claire knew that she was *judging* her. Claire started to get mad when Mrs. Astor spoke.

“Actually, Claire, I’d like to see how you do these problems.”

This went back and forth for about five minutes, but Claire couldn’t get Mrs. Astor to see that it wasn’t that big of a deal if she didn’t start right now. She finally looked at the problem and was relieved when she knew the answer. “ x is 4,” she said.

Mrs. Astor blinked and smiled; the stupid woman was clearly completely surprised that Claire got the right answer. Good - the lady would finally realize that Claire wasn’t a total idiot.

“Good, but how did you get that,” Mrs. Astor pressed.

Claire sighed and rolled her eyes, secretly happy that she showed the lady up. She wrote down “ $1) x = 4$ ” on the otherwise blank piece of paper, and set her pencil down.

Mrs. Astor didn’t let up. “Claire, how did you get that answer?”

“They’re both a half,” she said, and tried to put as much of a “*duuuuh*” tone as she could into the question.

“Okay, that’s good.” Mrs. Astor scanned the problems and said “Try number eight.”

Claire looked briefly at the question: $\frac{-3}{x+5} = \frac{2}{-12}$, and the answer wasn't obvious. She'd need to ask her friend later, and was angry at Mrs. Astor. "What, me getting one right isn't good enough for you, you have to go find out that's hard, just to make me feel bad? I'm not good at math, is that what you want me to say?"

"No, not at all. You got that first problem, which means you're not dumb. You did it your own way, so that means you have some basic understanding of this. So I skipped ahead to find one that I didn't think you'd know because I figured it didn't make any sense to do problems that are a waste of your time."

"They're ALL a waste of my time," Claire said. "When am I ever gonna need to do something like that... negative three over... $x + 5$... whatever? When?"

"After all of the other algebra classes you have in high school? Maybe all the time, maybe never. You could probably make it through life pretty easily not knowing how to do it."

Claire didn't say anything; she wasn't expecting that answer.

"But that doesn't mean you don't need to know it or that you definitely won't use it. Maybe you get to high school and discover that you love working with animals and want to be a biologist, but oops, you gotta go back and take all that math which sounds awful because you think you're so far behind."

Claire could tell a few other students in nearby desks were listening, though they weren't actually looking at the two of them.

"And it's not like this is actually *hard* to do, just like walking isn't particularly hard. But if you've never done it before, it can seem like it's impossible, and most of the value of all of this comes in forcing your brain to think in ways that it's not used to. You get better at learning."

"But I don't *need* it, and I'm not *good* at it. It's a waste of my time."

"Trust me, Claire," I've seen worse. I've seen students in high school that wouldn't have gotten that first problem - those kids are actually bad at math. You, though... I'll bet you're not as bad at math as you think you are."

Claire snorted. "Yeah right, so why are *you* here?"

Mrs. Astor didn't bite. "Because I think you're afraid of math, not bad at it."

"Afraid... *afraid of math*?" Claire was irate. "I'm not afraid of math, I suck at it. I don't get it... these problems with x and π don't make any sense. They're stupid. I'm not *afraid* of math, I just don't think that way. And you haven't even been in here for an hour, and already you're telling me that I'm some sort of math coward? You're an *idiot*."

Mrs. Astor looked around at the few students watching them and said "why don't we have this little chat in the hall?"

Claire folded her arms. "You're not the teacher, you can't kick me out of class."

"I'm not kicking you out of class, and I'm not forcing you to do anything. But I'd like to talk about this without disrupting all of your friends."

Claire realized that the teacher would just make her go out with Mrs. Astor if she pushed it. "Fine," Claire fumed, and walked out ahead of her helper.

When they got into the hall, Mrs. Astor closed the door behind them. It was strangely, comfortably quiet. Mrs. Astor didn't look as angry as Claire expected.

"The reason I'm saying you're not bad at math is that I've looked at your testing and I've read what your teacher wrote about you."

"What did he write about me," Claire demanded.

"He's been making notes for a couple weeks on your classwork. According to him, you've missed about 30% of your class time doing things like going to the bathroom or the nurse.

Claire scoffed and denied it: "it hasn't been that much," but she wasn't really sure.

Mrs. Astor continued. "And when you were in class, you only had your stuff out about 10% of the time."

"So? I don't learn that way."

"So... you're avoiding it. You skip class, basically, and don't work on math when you're in it."

"That doesn't mean I'm afraid of it."

Mrs. Astor scratched her chin. "Maybe afraid isn't the right word. More like uncomfortable. Do you know what anxiety is?" Claire nodded. "Okay, well, there's general anxiety, where people are just sort of nervous all of the time, and there's trait anxiety, where people are nervous around certain things."

"What, like spiders?"

Mrs. Astor smiled. "Sort of. Trait anxiety is something that refers to things that most people aren't really afraid of, or where there's no reason to actually be afraid. Spiders can kill you. Math only feels like it can."

Claire wrinkled her nose. "So I have, what... math anxiety?"

"Maybe. But if we're going to figure that out, we have to make sure you're not bad at math first."

"But you just said I'm not bad at math."

"I don't think you are. But you have to actually do the math for me to see whether you're bad at it or not. Same goes for your teacher or your parents. That's why I'm here - not to spy on you, but to see if you're bad at math or just avoiding it."

"Yeah, but if I'm avoiding it, I should just be able to go and do it, but I'm telling you that I can't."

"Sure, but all of this math is based on math that you've probably been avoiding."

That's probably true, Claire thought.

"So I just want to work with you on some problems to see if you can learn and repeat what I ask you to do. That's all."

Claire thought for a few seconds. "So you're saying I can get better at it? That my anxiety can be fixed?"

Mrs. Astor smiled. "That's probably one of the easiest things I can help a student with," she said.

Section 1: About math anxiety

Chapter 1: Introduction

You are not that bad at math.

I mean, yeah, there's a small chance that you're actually "bad at math," but I'm playing the percentages here; an estimated three to six % of the population has a math disability (Gross-Tsur, Manor, & Shalev, 1996) while 20 percent of the population has math anxiety (Ashcraft & Ridley, 2005). Before your eyes glaze over, that means that about 75-80% of people that think they are bad at math actually aren't, they just think they are! Incredible, right?

So why are you shaking your head? After all, it doesn't make sense to you, or it's hard, or it seems kind of stupid, or you try and try and try but it never sticks, right? As a math teacher, I've had a lot of students come through my classrooms that have given me some version of one of these reasons. But here's the crazy part - while they may have been right in that it didn't make sense to them, or it seemed stupid, or it wasn't sticking - when it was anxiety, there was a good chance that the reason these things were true for them was not because of some deep down inability to do math, but because those students *believed* they were unable to do it. It's a big problem because the issue compounds itself; when students avoid doing that math they believe they're not capable of doing, they don't try as hard to learn any new concepts they're introduced to (Ashcraft & Ridley, 2005). And since they don't try hard, they don't learn the new material. Because math is so often sequential, most concepts are necessary for other math in the future. Thus, if a student doesn't learn something, it makes it harder for them to learn other things down the road that rely on that. In other words, it's harder to learn math that's based on math that's based on math that's based on math that's based on math that a person didn't learn well the first time.

Learning math is really like any stage of human development; we have to push our baby selves up before we can crawl, we crawl before we can pull ourselves up to balance against things, we have to balance against things before we can stand up, we have to stand up before we can walk, we have to walk before we can run, we have to run before we can jump, and so on.

In math, you have to count before you can add. You have to add before you can multiply. You have to multiply single digits before you can multiply several digits... and so on. At some point in school, the math gets difficult. What may surprise you (if you're one of those that think of yourself as mathematically inept) is that math gets difficult for *everyone*; even famous geniuses like Albert Einstein have struggled at some point (Randerson, 2006). The difference between most people that are "good" at math and most that are "bad" at math isn't some sort of natural ability, it's a belief that if it's difficult, it can still be mastered (Blackwell, Trzesniewski, & Dweck, 2007). Basically, the flip side of the "I'm bad at math, it would be a waste of time to try" is "This is hard, that's not right - I must be missing something. I'll figure it out." It is this mental attitude - not some sort

of math gene - that makes the people that are good at math stay good at math.

This is all a form of something called “learned helplessness.” Learned helplessness is the phenomenon where a person develops a belief in their inability to control a situation, even when that situation is actually under their control (Hiroto, 1974). Often times, someone that has had negative experiences with math may well develop a feeling that they are unable to do better, even when the research shows that intelligence (yes, even math intelligence) is fluid and can be improved by not avoiding criticism (Çetin, İlhan, & Yilmaz, 2014) and by working hard (Gunderson, Gripshover, Romero, & Dweck, 2013). This can be difficult - who wants to have someone tell them they’re doing something wrong? But the better able you are to brush your emotional reaction of that that criticism aside and absorb the actual meaning behind it, the better a student you’ll be (Gunderson, et. al, 2013).

In other words, regardless of whether you *are* bad at math, the fact that you *think* you are bad at math has an enormous impact on you actually *being* bad at math. And this anxiety, this fear of being around math, is all wrapped up in these feelings of learned helplessness; if you’re avoiding math, it’s obviously less work to just believe that you are incapable of doing it and give up rather than keep believing that you can do it but that it’s going to take some hard work.

Now, there are exceptions. Just like there are people with dyslexia, for whom letters don’t seem to read right, there are people with dyscalculia for whom numbers don’t really make any sense. There are all kinds of ways in which this happens, and that’s a fascinating topic for another day (and another writer), but that’s not why you’re reading this book. Basically, you know what numbers mean and represent. You know that 485 is bigger than 67. You know that if you add 3 and 2 you get 5. You know that if you buy a one-dollar candy bar and give the woman at the counter a twenty, you’d expect to get nineteen dollars back in change, and you know that that can be represented by $20 - 1 = 19$. For people that are bad at math, who are reasonably smart in most other ways, these can be challenging concepts. That’s what dyscalculia can look like (Gross-Tsur, Manor, & Shalev, 1996), and I’m betting that you don’t have dyscalculia.

Instead, what you most likely have is some sort of *discomfort* with doing math, some level of math anxiety. Maybe your foot starts to bounce when you’re doing a math problem, or noises start to seem louder when you’re working on math problems. Maybe you feel the need to get up and go to the bathroom during class... for a half-hour... every day. Maybe you get moody about the time math class starts, or maybe when you think about it, you get a little bit of a headache or a nervous flutter in your stomach. These are all traits having to do with a deep-seeded “flight” response that humans (and many animals) have developed over the ages. This flight response served our ancestors well when it came to running from saber-tooth tigers or rockslides, but it causes us problems when we feel a need to run away from something as mundane as doing math problems (Ahmed, Van, Kuyper, & Minnaert, 2013).

(Before I go any further, let's separate "general" anxiety from "math anxiety." People that are generally anxious feel those feelings in a lot of different places in life. People with math anxiety typically only feel nervous when they are doing math (Ashcraft & Ridley, 2005). If it's general anxiety that's the issue for you, this manual will help a little bit, but you're probably better off seeking other sources of advice.)

So you have this flight response, where somehow your brain looks at the sentence "Train A and Train B leave the train station at 4:15pm..." and lumps it into the same "NOPE" category as your caveman ancestor's brain might have reacted to coming home from the hunt to find an angry lion in his cave. Clearly, these are not the same threat level, and yet your response is not all that different; you go into resistance or avoidance mode. Most likely, you would skip the problem and just not worry about getting credit for it (caveman decides cave not that important). Sometimes, you get up and go to the bathroom (caveman goes for drink of water and hopes lion leaves). It's not like your brain is built for specific situations, after all - it takes the same handful of emotions and picks the best one (Ekman et. al, 1987).

So why do you react that way but other people don't? Maybe it was a bad teacher. Maybe it was doing poorly on a test that always kind of "stuck with you." Maybe it was those terrible timed math facts tests that you may have had back in elementary school - you know, where you get a big sheet with a hundred math problems and you have like five minutes to do as many as possible, and you have maybe 50 done after three minutes and you're feeling like you got most of them right when that annoying kid across the room slams his pencil down and says "done," and then the teacher walks up and looks at his paper and says "good job," and when he hands your paper back he says something along the lines of "you suck at this you should just quit school and become a hobo," (well, okay, not really, but that's how it makes you feel). Maybe it was some or all of these things, or maybe it was some other reason, long forgotten to you.

So you didn't learn it like you should have. That's okay; everybody learns at a different pace. People may tell you that how quickly you learn things is important, and sometimes it is. But what's much more important is that you learn it at all. If you're struggling in high school with algebra because you never got a good grasp of fractions, it's time to stop ignoring the fact that you don't know fractions, and start trying to learn them, even if the idea of that makes you want to vomit. (Try to avoid puking on this manual, though, because you still have a bunch to read.)

Thankfully, there are things you can do. This manual will help you get there. We'll talk about things you can do to ease your math anxiety, and then we'll go play with some math as both a way to practice that anxiety easement and learn some of the most important things that students often struggle with.

Yes, that means you're going to do math. Starting to get irritated? Then consider this

your first lesson: think happy thoughts. Your body and brain work together, and you have to make your body get on board if you're going to fight this thing. So take a deep breath, smile, and let's have a little discussion about math and why you're scared of it.

Chapter 2: What is Math Anxiety?

There are basically three criteria for having it. The first is the feeling of unease or dread when having to do any sort of math (Hembree, 1990). You might experience it through a need to avoid doing math, or a tendency to hurry through the math, or to try the math once and not think hard about it. Maybe you find it hard to concentrate on math although you can usually focus on your other subjects. People usually have some sort of physical reaction to it - a reaction (or reactions) that they may not notice. I mentioned the bouncing foot or the nausea, but there are a number of other possibilities. It could be tensed muscles, sweating, increased heart rate, headaches, faster breathing, etc.

Also, the anxiety is solely related to math (Hembree, 1990). If you feel anxiety about not only math, but history and English and science and hanging out with strangers and monsters under your bed, then you might have a more generalized anxiety or anxiety related to something broader than math (like school in general or being around a certain group of people that you associate with your math class). If that's the case, then focusing on treating math anxiety isn't going to be all that helpful - it would be like having gangrene and taking medicine to reduce the fever. You'd need to treat the source of the problem, not the symptoms.

Lastly, the anxiety has to be a separate issue from a math disability (Hembree, 1990). In other words, if you are in a calmed state, you should be able to do math fairly effectively, even if the math that you're doing is a little bit of a lower level than what would be expected of someone your age, and even if it takes you a little bit longer than you feel it should. If - even when you are calm - math is still problematic for you, then your math anxiety could exist *because* of a math disability, and thus remediating that disability should be the primary target of your efforts, not just the anxiety (although, if you have both, then some work on reducing anxiety is probably a good idea, too).

In order to get a feel for whether you have just math anxiety or a deeper math disability, we need to consider a couple things to help us distinguish the two. First off, if a teacher sits down with you, one-on-one, and shows you how to do a couple problems, can you usually then do that math on your own? If not, do you ask for clarification when you can't?

I'm not interested in whether it's easy or whether you can remember it later after you've been taught - that's a different issue - but whether you can understand the math when distractions are reduced and it's not as easy to avoid doing the work. If you have someone show you how to do it, are forced to do the work, and actually try and think about the problem, are you capable of doing it (even if it takes an uncomfortably long time)? If you can't, there's a better chance that it's an issue with a math disability. If you can, then it's probably more likely math anxiety.

Next, do you notice a physical reaction to math, thinking about math, anticipating math, etc.?

If you recognize those butterflies in the stomach, or the pattern of leaving the room, then you're definitely staring at anxiety. Whether it's just anxiety or it's rolled in with a math disability is more of a difficult question to tackle, but suffice to say if you are *feeling* anxious about math, you're probably experiencing math anxiety.

So why does the anxiety cause problems for actually understanding math when you are forced to do it? Have you noticed that it seems to take you longer? A lot of people confuse this delay with a lack of math ability, when the reality is that, surprisingly often, the problem is due to anxiety. The reason has to do with how your brain works, in something called working memory.

You know what memory is, and you're probably aware of the difference between short and long term memory. If you don't, short term memory is your memories of the stuff that just happened (Rose & Craik, 2012) - when you walk into a room and blank out as to the reason you went in there, you're trying to access your short term memory for what you were just doing. Next week there's almost no way you'll remember, because it doesn't get stored in your long-term memory. Long-term memory involves those things that stay in your memory for a much longer period of time (Rose & Craik, 2012) - like who your first kiss was with or what your address is or (tragically) the image of that pop singer you hate. Things that stay in your brain are hanging around in your long-term memory area, and aren't as subject to being "cleaned out" the same way that your short-term memories are; your brain only has so much storage space for the things that are happening right now, so it tends let go of things that it deems unimportant - information that's only used once, that you have no emotional connection to (Rose & Craik, 2012).

Beyond those two, though, there's a third type of memory called "working memory." This is the stuff that you're thinking about at the moment, all the stuff that your brain is concentrating on while you're doing things. It can be distinguished from short-term memory in that it is more than just memory - it's how your brain and memory interact with each other ((Rose & Craik, 2012). For example, if you're adding $234 + 489$, your working memory might at some point be storing the $4 + 9 = 13$ *fact* that you already knew, the carry-the-one *process* that your brain is using, the $1 + 3 + 8 = 12$ sum that your brain quickly figured out, and the -22 part of the answer that your brain is only holding long enough for you to write the answer down. Your brain has a limited capacity for all of this, and while there's some evidence that it can be improved, people have varying ability to hold information in their head at one time (Ashcraft, 2007).

Imagine that you gave that $234 + 489$ question to someone who was wrestling a bear. You might expect that they would probably not be able to answer it very easily. Why? Because working memory is limited (Rose & Craik, 2012). Since wrestling both a bear

and a math problem requires using their working memory for multiple tasks, trying to concentrate on both the problem and dodging massive sharp-clawed swipes would make it difficult to do either one effectively.

So working memory is basically the information a person is processing at any one time (Rose & Craik, 2012). Because it is limited, anything that interferes with your ability to focus on math can reduce your ability to do the math and make you feel like you don't understand it.

So think of your working memory as like an empty glass. When a person starts a math problem, they have to put that problem itself into their working memory, which is like pouring water into that glass. Then they start thinking about that problem, which is more water in. If they're recalling procedures for solving that problem from their brain, that's more water. If they have to use a calculator, that's more water. For more complex problems, it's easy to see that that glass can get filled up pretty quickly.

And here's the big problem - for people with math anxiety, *that anxiety is like water that's in that glass all of the time* (Ashcraft & Krause, 2007). So as someone with math anxiety starts working on a problem, they start using their working memory just like everyone else, but because that glass is already half-full (or half-empty) they run out of space more quickly, and find themselves forgetting little bits of information that are relevant to the problem (Ashcraft & Krause, 2007). It then becomes an exercise in frustration to try and put all that important stuff together when so much of the brain space necessary to do that is taken up by the anxiety and irritation that constantly gnaws at them. It's no wonder they think they're bad at math!

So how do you get better at the anxiety? There are few different ways, but the technique that has been shown to be most effective is called Systematic Desensitization (Hembree, 1990). Sometimes the process is called "flooding," and it basically works like this - if someone is afraid of spiders, you have them force themselves to work with spiders, albeit gradually. Just as you wouldn't take someone afraid of spiders and put them in a giant enclosed barrel full of spastic hairy tarantulas, it doesn't help to just give you a page of math problems and say "now breeeeeathe."

The other way to make the math easier on you, unfortunately, is to practice, practice, practice, especially with skills that can be made to require very little thinking like multiplication facts or algebra problem-solving steps. Because of that working memory issue, using less brain power thinking about smaller things (like multiplication facts or algebra tricks) will free your working mind up to keep a better handle on how all of that information fits together.

In this sense, think of your brain as a juggler. Have you ever wondered how jugglers can keep ten balls in the air while you struggle with three? They practice, and eventually those movements - catching the ball, rolling the arm, flicking it up in the air with the wrist

- become rote, to the point where they aren't even thinking about their arms, they're thinking about what they're saying or about little adjustments to their routines. Our brains kind of work the same way - they keep a bunch of information "in the air" and deal with that information as it needs to be dealt with. This set of skills - how the brain manages the information that it holds and uses - are called "executive functioning" skills, and no matter how good or bad the executive functioning of a brain is, it can *always* be helped by practicing the easier material.

So you need to get to the point where your math is the same way, where you aren't even thinking about those little intermediary steps in an algebra problem, or about the rule for multiplying fractions. The only way to get there is to practice and become comfortable with them. This book will give you the opportunity to do some of that, and the goal is for you to get better, but then we return to that pesky math anxiety issue - after all, if you're avoiding doing math because you're nervous about it, how realistic is it to ask you to do a whole bunch of math simply to get better at it? If it was as simple as telling yourself to just "go do it," you would have done it already, no?

So that's why we're dealing with the math anxiety first, or more importantly, with strategies for dealing with math anxiety. In the context of doing that, you will practice, and after we help you find ways of getting rid of that pesky math anxiety, you can go practice, practice, practice on your own without always having that brain-glass partially filled with water.

Chapter 3: Calming Anxiety

So you have Math Anxiety, and now you know what that is. What in the world are you supposed to do about it? The answer, unsurprisingly, is to do math! But, we're only going to do the math once you're ready (and you will be).

So we're going to have you start by solving a simple equation. Pencil at the ready!

The following should take you about 20 seconds to solve:

$4x - 3 = 4 \log_3(\sin(x^3))$ (Use the space below as a scratch pad)

I'm going to guess that you didn't give that more than a second's worth of thought. Why? Were you just on a roll with the reading and didn't feel like switching tasks, or does the sight of that equation make you want to run? (Note: that was a really hard equation and I didn't actually expect you to get it done.)

Let's try it again, but this time, I want you to focus on your physical reaction. What is your first thought when you look at a math problem? Is there a churning in your gut? Is it a tapping of your knee? Whatever it is, that is the first thing we actually want to work on, albeit with a simpler problem.

On the next page, you're going to see a few math problems that should *actually* be easy (no - for real this time!). Again, if they're not, then the issue is probably less about math anxiety and more about a math disability. It will actually make this exercise a little more effective if you can get someone to make you do the problems on the next page, but if you are all that you have, then you'll have to roll with it.

Now, close your eyes (after you're done reading this, of course), take a deep breath, flip the page, open your eyes, and begin on the first problem.

Problem 1:

a) Two trains start at a train station and travel in opposite directions at an average speed of 30 miles per hour each. How far apart are they after 2 hours? (actually spend some time trying to answer this, please)

b) Try and think about your emotional reaction(s) to having to solve that problem and note any of those below.

Problem 2:

a) John goes shopping and buys five cartons of milk for \$3.29 each. How much did he pay? (Actually try and answer this, please)

b) What was your first instinct when when you finished reading those two sentences (note it below)?

More important than the math that you did on the previous page was how your mind and body reacted to having to do it. Did you twitch? Did your heart flutter a bit? Did you get the tiniest bit sick? Did you want to skip the problem outright and keep reading? If you had someone there with you, were you a little annoyed that they were making you solve the problem?

If you're going to get past the anxiety, the first step is to be able to recognize those symptoms, because you have to start by controlling them. People like to think that they're perfectly reasonable, rational creatures, but the truth is that we are very often slaves to our emotions. When you tried to do those problems, how were your emotions affecting you? Do you see those same emotions emerging in class or on homework?

There is a relatively new theory in the field of Psychology called Mindfulness, which is the theory that "experiencing the present moment nonjudgmentally and openly can effectively counter the effects of stressors." (Hoffman, Sawyer, Witt, & Oh, 2010) In other words, being aware of your surroundings and your own reactions to those surroundings can help you reduce your impact that your stress has on you. I'll be honest - it sounds a little hokey. But I think it also works because when if you force yourself to stop and think about your own responses, you're replacing your natural reactions with something else much less unproductive. Behavioral scientists might call this classical conditioning; pair some sort of triggering event (called an unconditioned stimulus) with a desired response to that event (called a conditioned response), and you can replace your natural reaction (called an unconditioned response) with that conditioned response (Treanor, 2011) In other words, if your first response to the onset of anxiety is to do whatever those anxious thoughts are telling you to do, then you can alter how you respond to that anxiety by simply training your brain to pause before carrying out that panicky action! It's like lifting weights - your math-encountering brain is not strong enough to avoid rolling right into whatever it does when that anxiety comes on, and you have to actually work on strengthening your brain's ability to resist those tendencies if you want to get rid of them.

Plus, research has shown that Mindfulness can be an effective technique for mitigating anxiety disorders, so there's that going for it (Hoffman et al., 2010).

So we're going to try the math problem thing again, but this time, I want you to expect those reactions, and I want you to just kind of mentally "stand back" and observe yourself having those reactions. The ideal response to that will be an amused "huh," but we'll also take simply noticing them.

So off you go to the next page. Pencil in hand, deep breath, close your eyes, and turn the page...

Problem 1)

If I throw a baseball, and it takes 1.4 seconds to get to the top of its arc, how long will it be in the air, in total?

(Remember, as you do this problem, focus as much as you can on what you're feeling)

Problem 2)

Solve for x: $\frac{3}{x} = 12$

(Remember, as you do this problem, focus as much as you can on what you're feeling)

What were your reactions? Did you notice your frustration? Did you feel like you had any control over it? If you did, then great! If not, then keep trying when you're working on future math problems (especially those after this chapter).

I'm hoping that you're starting to see where I'm going with this - fully mastering any type of anxiety will obviously take time because it's most likely built itself up over a period of years. Undoing that can't happen in a day. But my hope is that you can take some of these tips and continue to practice them on a regular basis, and help reduce the anxiety yourself. Or get someone you trust to help, someone that WON'T put you down when you make a mistake, someone that can be patient with you as you try to learn this math (because otherwise they could just make the anxiety worse). Whatever you do, don't give up on yourself, because that's how anxiety wins. That's what you've been doing for so long, and that's what you need to stop. Keep going, work hard, and I *promise you*, you will get better.

Section 2: Common math deficits

Introduction

While there are a great number of math concepts that are good to know, teaching you how to do all of them would be impossible to do in this manual (talk about blowing up your anxiety!). However, in my years of experience as a high school math teacher, I've seen that a lot of students haven't learned how to do a few certain things that they keep coming in contact with, which I expect would be extremely frustrating, confusing, and a probable source of anxiety. These include:

1. Knowledge of basic math facts
2. Fractions
3. The equals sign
4. Negative numbers
5. Parentheses
6. Variables and Story problems

I suspect that, if you're reading this, one or more of these areas may be difficult for you, but you should be telling yourself that you *can* learn them. Your brain is smart enough to handle them, trust me - it just may take a little work, that's all. At this point they may be anxiety-producing, but that doesn't mean that you can't learn them, and coming from a math teacher: they are *very* important in later math!

I don't necessarily expect you to read through the rest of this manual; mostly, the first Section was the focus. However, if any of the above are difficult for you, then what a perfect time to practice the anti-anxiety techniques you already read about!

So take each of the following sections as you need them, and don't forget to breathe!

Best of luck!

Chapter 1: Basic Math Facts

If you're going to be successful in your future math classes, you have to know your basic math facts. Yes, I know... you could use a calculator, but as you read earlier it taxes your working memory, and reliance on a calculator acts as a barrier when you need to know the fact but don't have a calculator available (or don't feel like pulling out your phone to scroll through the apps).

So you need to learn them. Can you get by without knowing them? Sure, but then again, you can get by in life by just finding a shack in the woods and hunting squirrels. If you want to *thrive*, however, you need those facts.

Don't freak out if you don't know them, by the way; I've taught a lot of students that didn't know their basic facts, so you're not alone. And fixing this isn't all that hard - all it takes is repetition.

So get a set of flash cards for the facts you need (addition, subtraction, multiplication, and/or division). Alternatively, get yourself a set of 3x5 cards and write the one fact on the front and another on the back. Now, start drilling yourself on these over and over until you learn them. There is no simpler, easier, and more consistent way to learn these things (Duhon, House, & Stinnett, 2012).

So how should you go about studying these? Well, no two people work the same, so I can't really tell you. Some people like scheduling things out, while others like doing it when they feel like it. The problem with scheduling is how you cope when things aren't progressing as planned (which could increase anxiety). The problem with doing it when you feel ready is that you may never actually feel ready. Some combination of the two is good, but the most important thing is to spend time trying to find what works for you and ensuring that it actually *does* work.

Also, find your motivation. Find a reward for yourself and put someone else (that you trust) in charge of giving you that reward, per your instructions. Maybe you give your friend your phone and that person has to acknowledge that you memorized a set of facts before you get it back - that sort of thing. Make your "now" self write checks that your "future" self has to cash; it's a lot easier to get things done when you trap yourself into having to do it.

Tips:

1. At first, work on your own. Having to worry about the opinion of the person you're working with could end up hindering you; you have anxiety, remember? Only after you know the facts cold should you run them through with someone else.
2. Separate the facts you need to learn into groups, called blocks, based on the type of fact (+, -, /, *) and the first digit. Name those blocks by their operation and first number;

- 5x1, 5x2, 5x3, 5x4, etc., are the “times-5’s.” Put a rubber band around each separate block, and store them in a place where they’re not going to get all bent up.
3. Start with what you’re sketchy on. If you think you *kinda* know your times-4 facts, then start with those. It’s better to review those facts and build up your confidence than it is to continue to be okay with just knowing them pretty well.
 4. Learn them in blocks per day. Don’t put too much on yourself in one day. Maybe on the first day you start with learning times-5’s. Don’t include add-5’s or times-8’s, because that could confuse you.
 5. The first time through the cards, be ready to give yourself about 5-8 seconds to think through the problem. Don’t take too long, but don’t push yourself too hard, either.
 6. If you’re sure you know the answer, say it out loud. If you only *think* you know, then check your answer first. Either way, check your answer. Put the answered card into the back of your pile and go to the next one.
 7. Go through the pile at least three times. The third time through, you should notice that you’re getting them faster. You may not be getting them all yet, but don’t get too frustrated; it may take a while.
 8. Keep going through your pile until you are nailing each one of your facts, saying them out loud and getting them *all* correct.
 9. Put that block away for the day.
 10. The next day, pick that block back up and repeat this process. It should go faster, but it’s important that, even if you’ve moved on to a new block, you repeat this process anyway. On day 1 you were able to store all that information comfortably in your short-term memory, but on day 2 you’re doing it to make sure that it got imprinted into your long-term memory.
 11. You should do each block until you can do it without error, the first time through, for at least three days in a row. Then you can be confident that it’s stored well.
 12. Don’t work on more than three blocks in a day. You’re not trying to win a race here, you’re running a marathon. As frustrating as it may be - and it might take many weeks or months to get these all down - force yourself to take your time. Accuracy is your goal, not speed. You don’t want to confuse your brain by trying to cram too many facts in there at once, because that will just make the process take longer.

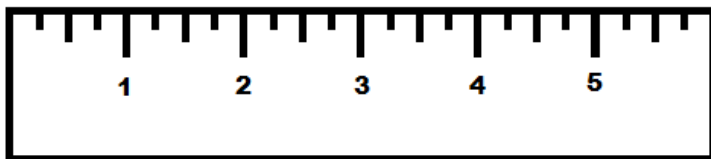
If you stick to this method, you should eventually get there. Again, make sure someone is helping you, and make sure you’re continuing to reward yourself and remind yourself of how good it feels to nail those facts each time through the order. And pay attention, too, to how much it’s helping at school. It will be a great feeling.

Chapter 2: Fractions

A great many people find fractions annoying, but fractions are one of the most important predictors of success in algebra (Booth & Newton, 2012). In other words, if you're going to continue to do well, you have to understand them; you shouldn't just default to punching them into the calculator and changing them to decimals.

So what is a fraction? Well, you may have heard "parts of a whole" as a description, and that's a pretty good one, but it turns out that looking at it just that way may not be the best way of looking at it (Tzur, 1999). Instead, look at it as a place on a number line, and try to think of it as if you were looking at a ruler.

You are probably familiar with a half of something as being $\frac{1}{2}$. What you may have been taught is that when you have half of something, you have one out of two possible. Again, that's a limited explanation, but it works for what we're doing.



Unfortunately, we still deal with inches in the U.S., so we're going to use an inch ruler here. Each big mark represents one inch, and is numbered (this is obviously not to scale). The next largest mark represents a half-inch. Given that, you'll notice:

Each inch is equal to 2 half-inches.

In other words, each half-inch is 1 part out of 2 in a whole, or $\frac{1}{2}$.

If you look at the smallest marks, four of those little distances makes up an inch.

Those are a quarter of an inch, or $\frac{1}{4}$.

In other words, each one is 1 part out of 4 in a whole.

Note: You may be starting to feel a little anxious here - there are a lot of numbers and a lot of information to hold in your head. If you have to put the book down for a second and focus on your reactions, trying to calm them down, then that's really the whole point of this exercise, so don't feel bad about doing it. But please come back!

But crazier still is that you can have a fraction like $\frac{7}{4}$, where there are more parts than are in a whole. This is confusing and irritating to a lot of people and probably why it's called an *improper* fraction; how can you have more parts than are in a full amount?

The answer is to look at it like here on the ruler: 7 parts out of 4 is simply 7 of those quarter-inches (since each quarter inch represents one out of four).

Counting from the left, you get 3 notches past the 1, which means you have 1 whole inch and three quarter-inches.

This is why you might rewrite that fraction as $1 \frac{3}{4}$.

How do you convert? Simply divide the top of the fraction (the numerator) by the bottom of the fraction (the denominator).

7 divided by 4 gives you 1 with a remainder of 3.

That remainder (3) goes into the numerator (the top)

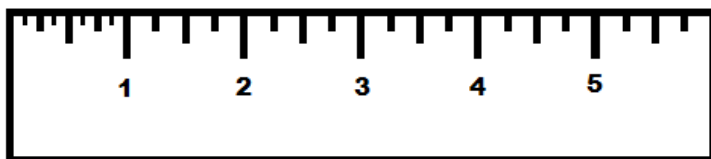
The 4 that you *divided by* goes into the denominator (the bottom)

You get 1 whole inch plus 3 out of the 4 notches through the next inch, or $1 \frac{3}{4}$.

Multiplication

So what about multiplication? In fact, what does it even mean to multiply $\frac{1}{2}$ by $\frac{3}{4}$? Well, think back to your “groups” analogy. What would it mean to to see half of a fourth?

Let's go back to the ruler:



Each of those itty-bittiest lines to the left of the 1 divides each of those quarter-inches in half.

In other words, each is half of a quarter, or $\frac{1}{2}$ of a $\frac{1}{4}$.

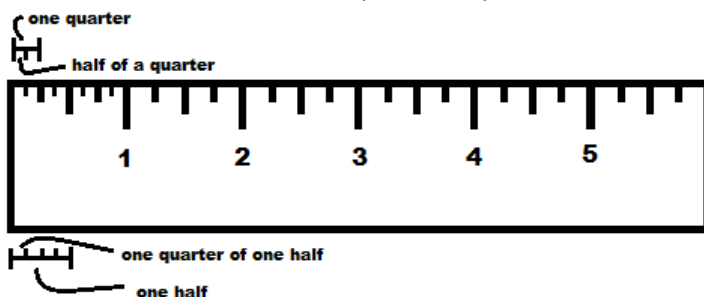
How many of itty-bitty distances are there? If you count them up, you get 8.

So each little notch is 1 out of 8, or $\frac{1}{8}$ of an inch.

Note: Feeling stressed? Take a breath and/or a break. Here's a little joke to ease your tension - what's orange and sounds like a parrot? A carrot.

So we get $\frac{1}{2}$ of $\frac{1}{4}$ giving us $\frac{1}{8}$. This is actually multiplication!

You can either look at it as half of a quarter or a quarter of a half:

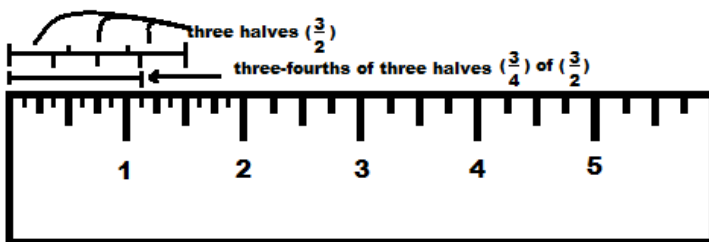


When you multiply, you break each of the parts in the whole up into more equal parts. In other words, you multiply the denominators together:

there are 2 parts for each of the 4 parts, so there are 8 total.

What if the numerators aren't 1?

Take $\frac{3}{2} * \frac{3}{4}$:



You'll notice that the answer ends on an eighth of an inch, and if you count those eighths, you get 9. You may also notice that if you multiply the numerators (3 and 3) you get 9, and if you multiply the denominators (2 and 4), you get 8. The answer, then, is $\frac{9}{8}$!

So the rule for multiplying fractions is:

Multiply all of the numerators together to get the *new* numerator (aren't I funny?)

AND multiply all of the denominators to get the new denominator.

It's that simple!

Addition/Subtraction

For adding and subtracting fractions, you have to get them in the same denominator.

What would a half of an inch plus a quarter of an inch be? If you simply added the

numerators and denominators, you'd get 2/6. That clearly is not correct; if you break the inch up into 6 parts, it's *shorter* than a half-inch.

The key to remembering how to add and subtract fractions is to think of the denominator as a unit of measurement, like a mile or a gallon. 1 fourth plus 1 half is sort of like adding 1 mile plus 1 foot. You can't just get 2 miles or 2 feet, because the units are different. You have to convert.

You've probably been taught how to convert, but maybe not why. It's really not that hard! To start, though, consider three questions:

1) What happens when you take a number (any number!) and multiply it by 1?

Answer: Nothing! It's value doesn't change!

2) What happens when you divide a number (any number!) by itself?

Answer: You get 1! (3 divided by 3 = 1, 2.14 divided by 2.14 = 1, etc.)

3) What happens when you multiply any number by a fraction where the numerator

and denominator are the same?

Answer: You multiply it by 1, which means you don't change it!

Note: If you read the chapter on equalities, picture me pointing at the paper here and going "See?! Told ya!"

This can look a little confusing, but consider what's going on in the following statement:

$$\frac{3}{3} * \frac{2}{5}$$

Above you see (3 divided by 3) multiplied by (2 divided by 5). What is the answer?

Well, if you remember from multiplying fractions above, you'd get

$$\frac{3 * 2}{3 * 5} = \frac{6}{15}$$

This certainly *looks* like a different fraction. But if you punch both $\frac{6}{15}$ into a calculator, you'll get the same decimal. So you haven't actually *changed* the value.

It works because when you take by 1 (because 3/3 = 1.)

% and multiply it by 3/3, you're just multiplying %

So we can use this little trick to change denominators (i.e. units), much like we would multiply a number of feet by 12 to get inches or a number of gallons by 4 to get quarts.

If I have ~~some number~~ ^{some number} ~~and a denominator~~ ^{and a denominator} ~~again, think~~ ^{again, think} of it as a common unit).

Basically, I have to figure out what number both denominators can divide evenly into.

Note: Read that previous statement as many times as you have to before you understand it... it's really important.

(This can be tricky if you don't know your multiplication facts - another reason it's good to know them!)

The smallest number that they can both divide into evenly is 20.

5 (from the

20.

Multiply $\frac{3}{5}$ by $\frac{4}{4}$ to get $\frac{12}{20}$ (again, 4 divided by 4 is 1, so multiplying change the value).

We also multiply $\frac{1}{4}$ by $\frac{5}{5}$ to get $\frac{5}{20}$ (again, 5 divided by 5 is 1, so multiplying won't change the value).

Now we have $\frac{12}{20} + \frac{5}{20}$, or 12 twentieths plus 5 twentieths. Same unit!

We end up with 17 twentieths, or $\frac{17}{20}$.

Of course, if you only see fractions as "parts of a whole," the "denominator as a unit" thing would be a little foreign. But it works, and for some people, that's a major "aha!" moment.

We'll hold there for fractions.

Chapter 3: The Equals Sign

The equals sign is not taught well (Knuth, Alibali, Hattikudur, McNeil, & Stephens, 2003). In elementary school, you were probably given problems like this:

$$3 + 4 = \underline{\quad}$$

This seems rather harmless; the answer is 7, and there's nothing actually wrong with writing it this way. Unfortunately, because students learn to "associate the equals sign with the arithmetic operations performed to get the final answer," the more accurate way of looking at the equals sign is ignored as students go through middle school and beyond (Knuth, et al., 2003)

As an example, look at the following two statements:

$$2 * \square = 9 \quad \square - 13 = -39 \quad (\text{Knuth et al., 2003})$$

These two statements have the same value for \square . If you have been through algebra you (hopefully) know that all that happened was that 9 was subtracted from both sides. This didn't actually change the value of \square , so \square has to be the same in both.

Students that have an understanding that the equals sign doesn't necessarily imply a sort of left-side dominance typically have an easier time in algebra (Knuth, et al., 2003). Why does this matter?

Substitution

The main reason that this matters is that students can get stuck on the "do something to the left to get the answer" train. In the example above, students that realize that an equals sign simply means that both sides are the same (compared to students that see the equals sign as something that requires solving) were more likely to notice that the only difference between those was that the 9 was subtracted from the second one.

So how does this affect you? Well, it turns out that there's this whole "substitution" thing in algebra that's really, really important. Look at the following problem:

Solve for x:

$$2x + 8 = \sin(x)$$

$$\sin(x) - x = 12$$

Now, you probably haven't been taught how to solve these. But this is where the whole substitution thing comes into play. See that $2x + 8 = \sin(x)$ equation? What that *a/so* says is that $\sin(x) = 2x + 8$.

So what? Well, if $\sin(x) = 2x + 8$, then $\sin(x) - x = 12$ can be rewritten as $2x + 8 - x = 12$! After all, that equals sign says that $\sin(x)$ and $2x + 8$ are equal to each other, doesn't it? And I'll bet that the $2x + 8 - x = 12$ is a lot easier for you to solve!

I use that example because it's a pretty clear demonstration of how the equals property matters, but that's not the only reason. The second issue is that when we translate from story problems, we can get hung up on equalities and forget the commutative property of equality. What is that? Take the following statement:
 "The castle is made of rocks."

You were probably taught that "is" means "equals," and this is a pretty good general principle, but it's also a trap. To explain why, consider that the following two statements are the same:

$$2 + 4 = 6$$

$$6 = 2 + 4$$

Hopefully that's obvious (if not, it might be a sign of true dyscalculia). Regardless, it doesn't quite work the same when you consider language. In other words, "The castle is made of rocks" only means the same thing as "Made of rocks is the castle" if you're Yoda. And even then, it's because of the grammatical structure, not the logical equivalence.

See, when we use "is," we usually use it to state that the second part is a property of the first part. In other words, if I were to say "Jacob is awesome," I would be saying that the word "awesome" describes "Jacob," but not the other way around - if someone asked me what "awesome" meant, it would be silly of me to say "awesome is Jacob." But in math, $2 + 4 = 6$ means *both* that 6 describes $2 + 4$ *and* $2 + 4$ describes 6. These aren't properties of each other - they are two things that have the same numerical value, and which one is written first is irrelevant.

Why does this matter? Not only do a lot of the algebra problems that you do will involve this substitution trick, (which, in my experience, it's a very strange leap for a lot of students), but there are a lot of things where understanding the idea of equivalence is vital for understanding what's happening.

Take, for example, the following fraction:

$$\frac{3}{4}$$

We can raise that to $\frac{6}{8}$. Why? Well, if you multiply $\frac{3}{4}$ by $\frac{2}{2}$, you get $\frac{3 \cdot 2}{4 \cdot 2}$, which is $\frac{6}{8}$.

But how come we are just allowed to multiply by $\frac{2}{2}$? I can't just take $\frac{3}{4}$ and multiply it by anything I want, can I? Well, remember what happens when you multiply a number by 1? Give it a second...

That's right: multiplying a number by 1 doesn't change it! 6 times 1 is still 6, 84 multiplied by 1 is still 84, -243.3825 multiplied by 1 is still -243.3825, and so on.

So if $\frac{3}{4}$ times 1 still leaves $\frac{3}{4}$, then $\frac{3}{4}$ times *anything that is equal to 1* will still give you $\frac{3}{4}$. What's equal to 1? Oh, I don't know... maybe $\frac{2}{2}$? Yep, anything divided by itself is 1! And since multiplying a number by [1] doesn't change it, then multiplying a number by [anything divided by itself] doesn't change it, either (since anything divided by itself = 1)

In other words,

$$\frac{3}{4} * 1 = \frac{3}{4}$$

so $\frac{3}{4} * \frac{2}{2} = \frac{3}{4}$ (since $\frac{2}{2}$ and 1 mean the same thing)

But, we also know that $\frac{3}{4} * \frac{2}{2} = \frac{6}{8}$.

So $\frac{3}{4}$ has to be equal to $\frac{6}{8}$.

When you fully understand that the equality means more than just "here's the problem, now operate on me until you get what your teacher wants," and in fact means "here are two things that are equal, figure out what can you do with this information," it changes your perspective on math. It's a subtle shift, but if you try to look at your algebra problems as if they are puzzles that follow certain rules - rather than chores - you may find that they tend to make more sense, and that they're a little more fun than you thought.

Chapter 4: Negative Numbers

Negative numbers are a pain in the butt. They are. Even though I'm a math teacher I kind of hate them a little bit, because I've made countless mistakes with them - either forgetting to include them, or where to include them. But that doesn't mean I just ignore them and hope they go away! Nope, I have to keep using them, because they can be really important.

Why are they important, you ask? Let's say, for example, that you own a company that sold \$250,000 worth of stuff last month, and paid \$325,000 in overhead (costs). If you take \$250,000 (what you made) and subtract \$325,000, you'd get -\$75,000. That represents your net profit. Notice the difference between +\$75,000 profit and -\$75,000 profit; the -\$75,000 is actually a loss! Hopefully you can see how that negative sign makes such an enormous difference in how you view your month.

If you go into any business or mathematical field, you have to use negative numbers. They're a part of life. Many people try to get by on mentally turning a situation where they'd add a negative into subtracting a positive - in the situation above, subtracting the \$325,000 from the \$250,000 is really the same as adding a -\$325,000 - but you really should learn both ways. So let's dig into them!

The Hole Picture

What does a negative number mean? It could have several different meanings, actually, but the best analogy is with dirt. Now, this isn't perfect - algebra, as an example, can force you to deal with weird uses of negative numbers that don't make any sense - but let's use the analogy of dirt anyway, because all we're doing is showing how numbers work, and whether the numbers numbers you're playing with actually mean anything or not, they still work the same. Now think of weighing that dirt. You can probably picture what 50 pounds of dirt looks like - it would look like a mound of dirt. But what would -20 pounds of dirt look like? Turns out, it looks like a hole in the ground.

Look at the following equation:

$$20 + (-20) = 0$$

This equation could be used to represent that piece of information above. If you take 20 pounds of dirt, and add it to a hole that is *missing* twenty pounds of dirt, then you have a nice flat surface, with no dirt missing and no mound.

So the weird thing about negatives is that they aren't values, but *missing* values, *lost* values, *owed* values, etc. In the equation $30 + (-20) = 10$, if you start with 30, and add a "missing" 20, you'd have 10 left over. Think of it like debt - if you have \$30 in your

wallet, you'd like to think you have \$30. But if you owe someone \$20, that debt is like having this -\$20 thing hanging over your head, so really, you only have \$10.

Note: You may be thinking here, "but I have \$30 because I can spend it!" This belief, though, is what gets people into major financial problems. Yes, you have \$30 that you can spend, but if you don't factor that -\$20 to how much money you see yourself as having, it WILL cause you problems down the line when the people that you owe money to show up to collect that cash.

Negative addition = Subtraction

You can probably see why $30 + (-20) = 10$ can be rewritten as $30 - 20 = 10$. Both statements are true, and both are really close to meaning the same thing, so we kind of use them interchangeably in algebra. To put it briefly: *adding a negative number is the same as subtracting the positive version of that number.* You've probably been taught that $+$ - (plus-minus) should be rewritten as just $-$ (minus), and that's a pretty good way of looking at it. Why does it work? Well, think of the whole debt idea - adding debt is the same thing as subtracting value. If I have \$30 in my wallet and pay someone \$20, that's basically me having \$30 and adding a -\$20 loss. Either way, I have \$10 left.

Multiplying by negative numbers

There are no two ways around it - multiplying by negative numbers can get to be a pretty strange concept. Remember that multiplication is taking groups of something; 5 times 2 means "5 groups of 2 of something." So what would 5 times (-2) mean? Well, it means that you have 5 groups of (-2). $-2 + -2 + -2 + -2 + -2 = -10$. If you owe 5 people 2 dollars, that's like having 5 "values" of -\$2, or having a "value" of -\$10.

But what if you multiply a negative by a negative? How can you have negative groups? This trips a lot of people up, but look again at $5 * (-2) = -10$ again. Does it matter what order you multiply things in? 3 times 4 is the same as 4 times 3, right? No, it turns out, it doesn't matter what order you multiply things in (this is called the *commutative* property). So change the order of what you had above, and now you have:
 $(-2) * 5 = -10$.

This means that if you have negative 2 groups of 5, you have negative 10. Wait, what? Well, if you're *missing* 2 groups of 5 apiece, then you're really *missing* 10 total.

Apply the same logic to negative times negative numbers. Let's say you owe 10 people \$20. That's $10 * -\$20$, or -\$200 of value. But let's say that you *lose* those debts, as in all those people mysteriously disappear. You can represent that as *missing 10 debts of \$20*, or $(-10) * (-\$20)$. You've just gained \$200! So a negative times a negative is a positive! Or, when multiplying negative numbers, two wrongs totally make a right!

So the simple rule is this: if you multiply a number - positive or negative - by a negative number, you change that sign. Otherwise, don't.

Alternatively:

- 1) Two negatives multiply to get a positive, and
- 2) A positive times a negative gives a negative.

(Here's a tip: pick a two-line chorus from a catchy song that you don't like - you don't want to ruin a good song! - and replace it with those two rules. It might sound awkward, but you'll remember them.)

And, as a quick rule for when you are adding/subtracting,

- 1) If two numbers have the same sign, add, and if they have different signs, subtract.
- 2) The sign of your answer will be the same as the sign of the "bigger" number. (The bigger number is the one farther away from zero. This number isn't "greater," but it has a larger "magnitude.")

Let's take some examples! *(Try these before you read the answers. I wrote them in English because it always felt easier to me to avoid reading words than to avoid seeing numbers.)*

$$(-20) + (-40) =$$

We're adding, here, so we have to use the two adding rules. Since the signs are the same, we add twenty and forty to get sixty. The "bigger" number is negative forty, so the answer has the same sign. The answer, then is negative sixty.

$$(20) + (-40) =$$

Again, we're adding. The signs are different, so we subtract. Forty minus twenty is twenty. Since negative forty is the number with a greater magnitude and is hence the "bigger" number, the answer will also be negative. The answer, then, is negative twenty.

$$(3)*(-4) =$$

We're multiplying, so let's "start" with the first number. Three times four is twelve. Since we're multiplying by a negative, we change the sign of the first when we get our answer. Since our first number is positive, we change it to get a negative. Our answer, then, is negative twelve.

Now, you might be asking yourself, what if the numbers are reversed?

$$(-4)*3 =$$

We have a positive times a negative. Since the second number is positive, it doesn't change the first. Therefore, after you get three times four equalling twelve, you keep the sign. Since the sign is negative, the answer is negative twelve. It works either way.

Now, some of you may find this confusing. You may have learned other rules for doing negative numbers, and that's fine. If those rules work for you, then keep using them! Ignore what I just said if it makes things worse! But either way, you have to go back to practice these rules until you are intimately familiar with them, and then you have to get used to using them in other classes. If you keep working at it, and you keep telling yourself that you can, then you will.

Chapter 5: Parentheses

Parentheses screw everything up. You'll be hopping along through a math problem, and all of a sudden you see those stupid curved lines and the problem goes all to pieces. Having those things in there can make a math problem look a lot worse than it actually is, and getting used to how they work can take some time. Honestly, some people probably never get past them. But if you're going to be a decent math student, then you need to get used to them. (Yeah, you might want remember your anxiety reduction techniques when thinking about *that* fact.)

Order of Operations

If you're dealing with parentheses, you've probably been taught Order of Operations. There are a handful of different ways that these are taught, but the priority that operations get is always the same:

- 1) Parentheses
- 2) Exponents
- 3) Multiplication
- 4) Division
- 5) Addition
- 6) Subtraction

Now, Multiplication and Division should really be on the same line, and addition and subtraction should be on the same line, but we can set that aside for now. The problem isn't those buggers, because you've been dealing with them and even if you make mistakes, they probably make sense. No, the problem is always those stupid parentheses (and exponents, to a lesser degree).

A quick aside: why are operations done in this order? Well, it's like the alphabet - there's no real reason that A has to come before B, it's just that one has to come first, and it really, really helps if everyone agrees what to do in a particular situation. There's actually no reason that parentheses have to come first, but if $3+5(4+8)$ gave a different answer depending on where you were from, that would cause a lot of problems, so mathematicians just all agreed a long time ago that this was how it would work, and they've been doing it that way ever since.

So we have to deal with these parentheses. But the rules always seem to keep changing! So let's delve into those rules and clear things up. At the end of this I'll give you a cheat sheet that should help you, but math will be a lot easier for you if you commit these to memory and become intimately familiar with their use.

Parentheses around a single number

If you see a set of parentheses around a single number, that's typically because it's either some relic from an earlier part of a problem or it has a negative number in it (or your math book/teacher decided to have you practice with them there in order to get used to the them.)

As an example, look at the following:

$$3 * (4 - 8)$$

Order of operations insists that anything in parentheses is done first, so you have to subtract the 8 from the 4. This leaves you with:

$$3 * (-4) \text{ [since } 4 - 8 \text{ is } -4]$$

Notice that because of that negative, you should leave the parentheses there. It might seem harmless and easier to rewrite that as $3 * -4$, but trust me - that's a really bad habit to get into. I see students drop those all the time in the wrong places because (I suspect) they just don't like them. They're important - learn to trust them.

Plus, imagine that you had it written without the multiplication sign:

$$3(-4).$$

If you drop that parentheses, you might do so assuming that you'll know that $3 - 4$ means 3 times -4, but that's the funny thing about working memory... sometimes doesn't work, and when you drop those parentheses, all of the sudden $3(-4)$ becomes $3 - 4 = -1$.
Oops.

So don't drop the parentheses!

Distribution

So you've probably seen something like this:

$$4(3x + 8)$$

and have been taught that you have to multiply the 4 by both $3x$ and by 8. This is called "distribution" because you're distributing that 4 to everything inside the parentheses. Seems simple enough there, but it gets more complicated, of course.

First off, why does distribution work the way that it does? Why, exactly, is that 4 multiplied by the $3x$ and the 8 separately? To see why, think back to your groups analogy. This problem, then, says that you have 4 groups of $(3x + 8)$. In other words, you have:

$$(3x + 8) + (3x + 8) + (3x + 8) + (3x + 8)$$

[Note: We'll come back to this problem in a second, but I want to take you on a tangent off of the tangent we were just on. Consider the following:

$$(3 + 4) + (3 + 4) =$$

You can see that this becomes:

$$7 + 7 = 14$$

But $(3 + 4) + (3 + 4)$ can also be written as:

$$3 + 3 + 4 + 4 = \\ 6 + 8 = 14$$

What happened there? Well, when you're adding, it doesn't matter what order you add numbers in. It's the dual properties of associativity and commutativity, but basically it means that as long as there's only either multiplication OR addition (not a mixture), you can do them in any order.]

So back to $4(3x + 8) = (3x + 8) + (3x + 8) + (3x + 8) + (3x + 8)$
We can reorder these and rewrite it as:

$$3x + 3x + 3x + 3x + 8 + 8 + 8 + 8.$$

Since you have 4 $(3x)$'s and 4 (8) 's, this simplifies to:

$$4(3x) + 4(8).$$

So that's why distribution works the way that it does. If you have a whole bunch of a group, you have that whole bunch of each of the parts of that group. Make sense?

Adding/Division/Variables

Of course, all of this seems to say nothing about the following:

$$(3x^2 + 4x + 2)(2x + 5).$$

This is often where people quit in frustration and fear. Remember, though, we're *conquering* that fear. You have anxiety, not an inability to do math. I'm going to walk you through this.

See if you can follow my logic here.

1) $3x^2 + 4x + 2$ represents some number. We don't know what that number is, but it's a number. If x is 5, then $3x^2 + 4x + 2$ becomes 97. [$3(5)^2 + 4(5) + 2 = 75 + 20 + 2 = 95 + 2 = 97$.] If x is 3, then $3x^2 + 4x + 2$ becomes 41. If x is *any* number, then $3x^2 + 4x + 2$ is some other number. It may be a little weird to think of, but that $3x^2 + 4x + 2$ isn't just a set of variables and numbers, it represents an actual *possible* number!

2) Whatever that number is, we have $(3x^2 + 4x + 2)$ groups of $2x + 5$. Again, don't lose sight of the fact that $(3x^2 + 4x + 2)$ is a number, because the rules for numbers are the same regardless of whether or not they're written in code with x 's and stuff.

3) Since we have $(3x^2 + 4x + 2)$ groups of $2x + 5$, then we have $(3x^2 + 4x + 2)$ groups of $2x$ and $(3x^2 + 4x + 2)$ groups of 5. *(If you need a minute, take one. Recognize your symptoms and tame them! Then come back and, if it takes you 10 times, reread this step until you understand it. If you can't seem to get it, don't just move on and pretend it isn't a big deal - that's the anxiety talking and you're here to get rid of it - find someone that can help explain it.)*

4) Therefore, what we have is $(3x^2 + 4x + 2)(2x + 5) = (3x^2 + 4x + 2)(2x) + (3x^2 + 4x + 2)(5)$.

5) Great. Now what?

6) Remember that it doesn't matter what order you multiply things in. So if you have $(3x^2 + 4x + 2)(2x)$, then you can also write that as $(2x)(3x^2 + 4x + 2)$. Similarly, you can also rewrite $(3x^2 + 4x + 2)(5)$ as $(5)(3x^2 + 4x + 2)$

7) Have I changed the problem? That's for freshmen philosophy majors to decide, I suppose. The reality, however, is the following:

$$(3x^2 + 4x + 2)(2x) + (3x^2 + 4x + 2)(5) = (2x)(3x^2 + 4x + 2) + (5)(3x^2 + 4x + 2),$$

because *it doesn't matter what order you multiply things in. (again, read though this step slowly, and reread if necessary. There are a lot of numbers to process):*

8) So now I have $(2x)(3x^2 + 4x + 2) + (5)(3x^2 + 4x + 2)$. It's distribution *Inception*... we have to go *deeper*. *(You swallow that annoyance right now!)* Reset your thinking... it's almost like we're on a new problem.

9) If I have $(2x)$ groups of $(3x^2 + 4x + 2)$, then I have $(2x)$ groups of $3x^2$, $(2x)$ groups of $4x$, and $(2x)$ groups of 2. So we get the following:

$$(2x)(3x^2 + 4x + 2) = (2x)(3x^2) + (2x)(4x) + (2x)(2) \text{ and}$$

$$(5)(3x^2 + 4x + 2) = (5)(3x^2) + (5)(4x) + (5)(2).$$

Quick note because part of this confuses people... if I have $(5)(3x^2)$, does this become $(5)(3) * (5)(x^2)$? No! Why? Well, think of what multiplication means:
 $5(3x^2) = 3x^2 + 3x^2 + 3x^2 + 3x^2 + 3x^2 = 15x^2$. But $(5)(3) * (5)(x^2) = (15)*(5x^2)$, and now you can see that you would be required to distribute again, and things would very quickly get out of hand.

So that's an important math safety tip:

You can only distribute onto things that are being added or subtracted from each other.

In other words, when you multiply $(5)(3x^2 + 4x + 2)$, you're distributing the "5 times" onto the three things that are being added to each other.

But let's go back to where we were, shall we?

$$(2x)(3x^2 + 4x + 2) = (2x)(3x^2) + (2x)(4x) + (2x)(2) \text{ and}$$

$$(5)(3x^2 + 4x + 2) = (5)(3x^2) + (5)(4x) + (5)(2).$$

Notice what happened. You have 2x and 5 both being multiplied by $3x^2$, 4x, and 2, but you could also look at it as if you had $3x^2$, 4x, and 2 all being multiplied onto the 2x and the 5. Another little shortcut: multiply each term in the first parentheses by every term in the second, or vice/versa. A visual way to represent this is with the box method, which you may have seen before:

$(2x+5)(3x^2 + 4x + 2)$	$3x^2$	$4x$	2
$2x$	$2x(3x^2)$	$2x(4x)$	$2x(2)$
5	$5(3x^2)$	$5(4x)$	$5(2)$

Adding all of these together gives:

$$2x(3x^2) + 2x(4x) + 2x(2) + 5(3x^2) + 5(4x) + 5(2) =$$

$$6x^3 + \underline{8x^2} + 4x + \underline{15x^2} + 20x + 10 =$$

$$6x^3 + \underline{23x^2} + 24x + 10 \quad (\text{Note: I underlined and italicized only so I didn't lose you. I hope it helped!})$$

So the important rules to consider here are:

- 1) Don't drop the parentheses.
- 2) Multiply each term in the first parentheses by every term in the second, or vice/versa
- 3) Distribute ONLY onto things being added, not things being multiplied.

Chapter 6: Variabilization and Story Problems

As a teacher, I have a love/hate relationship with story problems. I love them because they help actually connect the math to the real world, but a LOT of students skip them whenever they can because they seem really hard, and that throws off my ability to actually assess whether my students actually learned the material. It's one thing to not know how to do it and another thing completely to kind of know how to do it, but willing to skip it and take the points off because you're not sure that you're doing anything but wasting your time.

Story problems don't have to be hard, of course - there are tricks that you can use to make them easier on yourself, and we'll learn those - but they do require more thinking because you aren't going to be shown every single situation where and how a new math concept can be used. I personally feel that too many people get so used to being required to do things a certain way in math that they just aren't taught what to do with the anxiety that comes right along with that uncertainty, and so they get in the habit of skipping the problems because teachers just don't want to have that fight all the time.

The most important habit you could possibly learn is to write down all important information, and variabilize it.

You shouldn't just *dive in* to a story problem. Sometimes you can, of course, but if you want to learn to be able to do the hard ones, you should get used to the idea that you need to write some things down first. What? Well, take every piece of information and name it!

Take this as an example:

Bob's dad is 42. Twelve years from now, Bob will be half his dad's age. How old is Bob right now?

This is a somewhat unusual example, because it's pretty information-dense. There isn't much about the colors of trains passing toward each other or anything; it's all numbers. That being said, the rule still stands - what pieces of information are we given? I know, I know... it's right there in the problem, so it feels like a frustrating waste of time to rewrite it, but that frustration is in no way unique to you - mathematicians have understood this so well that they invented the system of variables primarily so they don't have to write as much. So use them!

In fact, "variabilizing" information should be the first step in solving *any* story problem. You should get comfortable with translating what you see into math. In the story problem above, for example, the fact that Bob's dad is 42 years old should be written like this:

D = Bob's dad's current age.

$$D = 42.$$

Why did I write “ $D = \text{Bob's dad's current age}$?” Because I want to write everything down in order to make sure that I can follow my own logic. Again, remember that you shouldn't be relying too heavily on your working memory to keep all of this stuff straight, because your working memory is not perfect. Writing down “ $D = \text{Bob's dad's current age}$ ” should take you somewhere in the neighborhood of 10 seconds. Is that time really worth that much to you that you have to skip it? (The answer, in case you were wondering, is probably no.)

So you have:

$D = \text{Bob's dad's current age}$

$$D = 42$$

What about that “twelve years from now, Bob will be half his dad's age” part? Use what you have as a guide!

If $D = \text{Bob's dad's current age}$, what will his age be in 12 years?

Think about it a second....

...

$$D + 12!$$

(You might also have deduced that Bob's dad's age will indeed be 54, but let's ignore that for a second. You'll have problems where figuring that stuff out won't be so easy, so let's focus on how to do this with a problem where you can see any mistakes early.)

So we have:

$D = \text{Bob's dad's current age}$

$$D = 42$$

$$D + 12 = \text{Bob's dad's age in 12 years.}$$

What about Bob?

$B = \text{Bob's current age.}$

$$B + 12 = \text{Bob's age in 12 years.}$$

You're following both of those, right?

So let's take that “Bob will be half his dad's age” part:

“Bob” really means “Bob’s age in 12 years.” That’s translation from English to math, because, taken literally, Bob would be defined only as a single number, but we all know that Bob is far more interesting and complex than anything that his age in years could possibly represent.

So now we have “Bob’s age in 12 years will be half of his dad’s age.” Again, we can assume that it means “Bob’s dad’s age *in 12 years*,” because that’s what makes the most sense for the problem.

Note: if you ever need to clarify any of this, do not be afraid to ask your teacher. I always subscribe to the notion that it’s better to think you look like an idiot than to get it wrong and actually prove it. Plus, if your teacher is any good at all, they’ll appreciate you seeing something from a different viewpoint; those tests aren’t perfect.

So we’ve finally extended it to “Bob’s age in 12 years will be half of Bob’s dad’s age in 12 years.” Super duper! We have those pieces of information, and now we can write it as:

$(B + 12)$ will be half of $(D + 12)$

If you haven’t seen it before, a very common translation is to take “is” or “will be” or something definitive like that and turn it into an equals sign.

So now we’ve moved on to:

$(B + 12) = \text{half of } (D + 12)$

And “half of” simply means “ $\frac{1}{2}$ times”, so we have:

$(B + 12) = \frac{1}{2}(D + 12)$

And now we have our equation! At this point, solving the problem is just algebra that you’ve done before!

Let’s try another one...

Lafawnda wants to go visit her online boyfriend, Kip. She gets into a bus and starts riding from Hartford, Connecticut to Broken Bow, Nebraska, which is at 1570 miles away. Halfway across the country, at the exact same time, Kip wants to see Lafawnda, so he borrows his uncle’s motor home and begins driving that same trip in reverse, which is 1570 miles. If Lafawnda’s bus trip averages 40 miles per hour and Kip’s trip averages 65 miles per hour, when will they tragically and ironically pass each other on the road?

So here we have one of those overly-wordy story problems that can’t be done in your head. There’s nothing that necessarily indicates what direction we should go in or what type of math we should use, so we’re going to have to problem-solve this.

Thankfully, the problem creator (me!) didn't put in unnecessarily difficult complications like giving them different starting times. The point of these story problems, after all, is to get you to connect them to the material that you're learning, so they're usually reduced to overly simplistic situations in order to give you problems that you can solve. In the real world, of course, problems like this have lots of variables as well as differentials (which you may or may not learn about in calculus) that can more closely model situations like this with the trade-off of making the calculations and models harder to deal with.

So let's start by writing down our information. We do this by going word-by-word until we get to something that can actually be turned into math.

The fact that Lafawnda wants to visit Kip is relevant to the story, but it's not information that we can turn into math, so to speak. It's there to define the events and help us conceive of what's actually going on so that we can turn the rest of the problem into math, but it's not, in and of itself, math. The fact that she's on a bus isn't that relevant, either, so we move on.

We're told that the distance between Lafawnda and Kip is 1570 miles. We could call this piece of information whatever we want, of course, but more often than not you'll see "d" used to represent "distance," so we'll use d here.

In other words,

d = distance traveled

$d = 1570$

The names of the towns is obviously irrelevant, added there to give the problem some personality. You might be tempted to try and turn "halfway across the country" into math information, but it's not really all that relevant; again, it's there to add flavor to the problem. The same thing is true about Kip borrowing and driving a motorhome; it helps justify the difference in speeds, but that's about it.

The fact that Kip and Lafawnda both make the same trip is very useful information, because it simplifies the problem. If the two of them traveled different distances, we would have to use d_1 and d_2 (or d_L and d_K). Since the distance is the same, we'll just use the same variable.

Now we find out that Lafawnda's trip is 40 mph and Kip's trip is 65 mph. We can again use whatever variables we want, but when we talk about speeds, the most common convention is to use either v (for velocity) or r (for rate). Why should we avoid using "s"? Because if you write it, it looks like a 5. "s" isn't used as a variable all that often for precisely that reason.

[A quick aside... s isn't the only variable that's avoided. We don't usually use "e" or "i", because those actually represent numbers in the same way that π represents what it

does (3.14...). You'll probably also want to avoid using the letters f (because it's used in function notation), l (because it looks like a 1), and o (because it looks like a zero). Interestingly, t is often used for time, but it's distinguished from a $+$ when writing by giving it a curl at the bottom.)

So let's use v (for velocity!). We have two velocities, unfortunately, so we're going to use v_L (for Lafawnda's velocity) and v_K (for Kip's velocity). Again, you can use whatever variables you want, but it's good to realize that the more obvious you can make your variables, the easier it will be for whoever is reading your work (eg. your teacher) to follow your thinking.

So we have the following pieces of information:

d = distance traveled
 v_L = Lafawnda's speed
 v_K = Kip's speed
 $d = 1570$
 $v_L = 40$
 $v_K = 65$

Now, that may seem like all the information the problem gives us, but there's one more very critical piece of information that we haven't written yet... one that a lot of students don't even think to write. What is it?

"When will they... pass each other on the road?"

What is this piece of information telling us? It's telling us what we're actually supposed to look for! If you don't know what to look for, you have no hope of solving the problem! So let's try to variabilize this...

Uh...

They way to do this isn't obvious. So we have to start asking ourselves questions. The first thing you want to ask yourself is: What *thing* are we actually trying to find?

Basically, we're trying to find either time or distance, depending on how we read the problem. The word "when" is a clue that it's looking for time, but we often confuse distance and time in the real world (such as... "My school is fifteen minutes away from my house"), so maybe the problem is asking is for a distance. We'll keep both in mind, because the case is often that if you find one, it's pretty easy to get the other. That being said, I've always felt that distance is a little easier, so that's the one I'm going to be using.

So which of the following are we looking for:

- 1) When Kip's distance traveled equals Lafawnda's distance?
- 2) When the total distance between them minus Kip's distance traveled equals Lafawnda's distance?
- 3) When the total distance between them minus Lafawnda's distance traveled equals Kip's distance?

It's tempting to say 1, but remember that they are going at different speeds, so the faster driver (Kip) will go farther. The correct answer is actually *either* 2 or 3, since both of them give you the same thing! You could either get really frustrated at that fact, or you can realize that, sometimes, even if your method of doing a math problem isn't the same as someone else's, that doesn't necessarily mean you're wrong. This is the hardest part of the problem, though, and knowing the (Total) - (one person) = (another person) math trick would certainly help.

But what if you don't know? What if you are looking at the problem and still can't figure it out? A trick I like to teach my students is to plug some numbers in and see what happens. Sometimes when you start putting numbers into a situation, the correct equation magically seems to appear! We'll come back to this one, but I wanted to make sure that I'm not forgetting that it's very common to not know how to set up equations.

So let's take when the total distance between them minus Kip's distance traveled equals Lafawnda's distance.

The total distance between them = $d = 1570$ miles. This has already been mentioned. Do we know how far Kip or Lafawnda have traveled? Nope! So we variabilize them.

d_K = distance Kip travels

d_L = distance Lafawnda travels

And for reference...

v_L = Lafawnda's speed

v_K = Kip's speed

$d = 1570$

$v_L = 40$

$v_K = 65$

So now what? Are we done? This is where we get to the thing about plugging numbers in again. Let's assume that Kip drives for... oh, I don't know... 650 miles. What do we know about it? Well, if he's averaging 65 mph, he's been driving for 10 hours.

Oh, crud. We have to consider time.

t_K = time Kip travels

t_L = time Lafawnda travels

How did we know that Kip drove 10 hours? We have to think about what our brains did. Because $65 * 10 = 650$, we did:

$$v_K * t_K = d_K.$$

Hey, that looks like an equation! You might have seen the formula distance = rate * time before. This is just a different form of that.

We can therefore assume the same scenario for Lafawnda: $v_L * t_L = d_L$.

So let's go back to finding what we're looking for. The total distance traveled minus Kip's distance is the same as Lafawnda's distance.

In other words,

$$1570 - d_K = d_L.$$

Do we have either of these distances? No. But we do know that d_K and d_L each had separate equations:

$$v_K * t_K = d_K$$

$$v_L * t_L = d_L$$

So let's replace d_K and d_L with these other equations.

Now we get:

$$1570 - d_K = d_L \text{ becomes}$$

$$1570 - v_K * t_K = v_L * t_L$$

Cool. We know what v_K and v_L are, so we can plug those in.

$$1570 - v_K * t_K = v_L * t_L \text{ becomes}$$

$$1570 - 65t_K = 40t_L$$

Hmm... we have those stupid times. But wait! The problem said that they started at the same time. Therefore, $t_K = t_L$, since they have traveled the same amount of time! So let's just call both of them t .

Now we have:

$$1570 - 65t = 40t.$$

Add $65t$ to both sides:

$$1570 - 65t + 65t = 40t + 65t$$

and we get:

$$1570 = 105t$$

Divide both sides by 105:

$$\frac{1570}{105} = t = 14.95 \text{ hours.}$$

So they will cross each other's path after almost 15 hours on the road!

Now, this may seem like a really long way to solve a problem, but you'll only write a fraction of what you read here when you're doing these, and a lot of this will be done in your head. As you do more and more of them, then over time, just as with any skill, you'll find yourself improving and these things getting easier. But story problems are hard for a reason, and success isn't going to happen overnight.

The keys to remember for solving story problems are:

- 1) Variabilize your information, even the stuff you don't think you'll need.
- 2) Go almost word-by-word if you have to. Reading story problems isn't a quick endeavor. It may be frustrating, but isn't it more frustrating to keep getting them wrong?
- 2) Try making up numbers for unknowns to see what happens in order to put them properly into equations. Sometimes the problem makes a lot more sense this way.
- 3) Don't give up! Sometimes you won't get it the first time, and that's okay. One of the reasons you're taught math at all is that you're *supposed* to work yourself through logically difficult problems. It's a skill that will help you in life.
- 4) Write down EVERYTHING! More than once if you have to. It's better to write too much than too little; nobody's going to miss the paper you use.
- 5) Keep breathing. Your anxiety is trying to rule you - don't let it.

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